1.0 INTRODUCTION TO STRUCTURAL ENGINEERING

1.1 GENERAL INTRODUCTION

Structural design is a systematic and iterative process that involves:

1) Identification of intended *use* and *occupancy* of a structure – by owner
2) Development of architectural plans and layout – by architect
3) Identification of structural framework – by engineer
4) Estimation of structural loads depending on use and occupancy
5) Analysis of the structure to determine member and connection design forces
6) Design of structural members and connections
7) Verification of design
8) Fabrication & Erection – by steel fabricator and contractor
9) Inspection and Approval – by state building official

Ideally, the owner and the architect, the architect and the engineer, and the engineer and the fabricator/contractor will collaborate and interact on a regular basis to conceive, develop, design, and build the structure in an efficient manner. The primary responsibilities of all these players are as follows:

- Owner - primary responsibility is deciding the use and occupancy, and approving the architectural plans of the building.
- Architect - primary responsibility is ensuring that the architectural plan of the building interior is appropriate for the intended use and the overall building is aesthetically pleasing.
- Engineer – primary responsibility is ensuring the safety and serviceability of the structure, i.e., designing the building to carry the loads safely and ___________.
- Fabricator – primary responsibility is ensuring that the designed members and connections are fabricated economically in the shop or field as required.

- Contractor/Erector - primary responsibility is ensuring that the members and connections are economically assembled in the field to build the structure.
- State Building Official – primary responsibility is ensuring that the built structure satisfies the appropriate building codes accepted by the Govt.

1.2 STRUCTURAL DESIGN

- Conceptually, from an engineering standpoint, the parameters that can be varied (somewhat) are: (1) the material of construction, and (2) the structural framing plan.
- The choices for material include: (a) *steel*, (b) reinforced concrete, and (c) steel-concrete composite construction.
- The choices for structural framing plan include moment resisting frames, braced frames, dual frames, shear wall frames, and so on. The engineer can also *innovate* a new structural framing plan for a particular structure if required.
- All viable material + framing plan alternatives must be considered and designed to compare the individual material + fabrication / erection costs to identify the most efficient and economical design for the structure.
- For each material + framing plan alternative considered, designing the structure consists of designing the individual structural components, i.e., the members and the connections, of the framing plan.
- This course ***CE405*** focuses on the design of individual structural *components*. The material of construction will limited be steel, and the structural framing plans will be limited to braced frames and moment resisting frames.

1.3 STRUCTURAL FRAMEWORK

- Figure 1 shows the structural plan and layout of a *four*-story office building to be located in Lansing. Figure 2 and 3 show the structural elevations of frames A-A and B-B, respectively, which are identified in Figure 1.

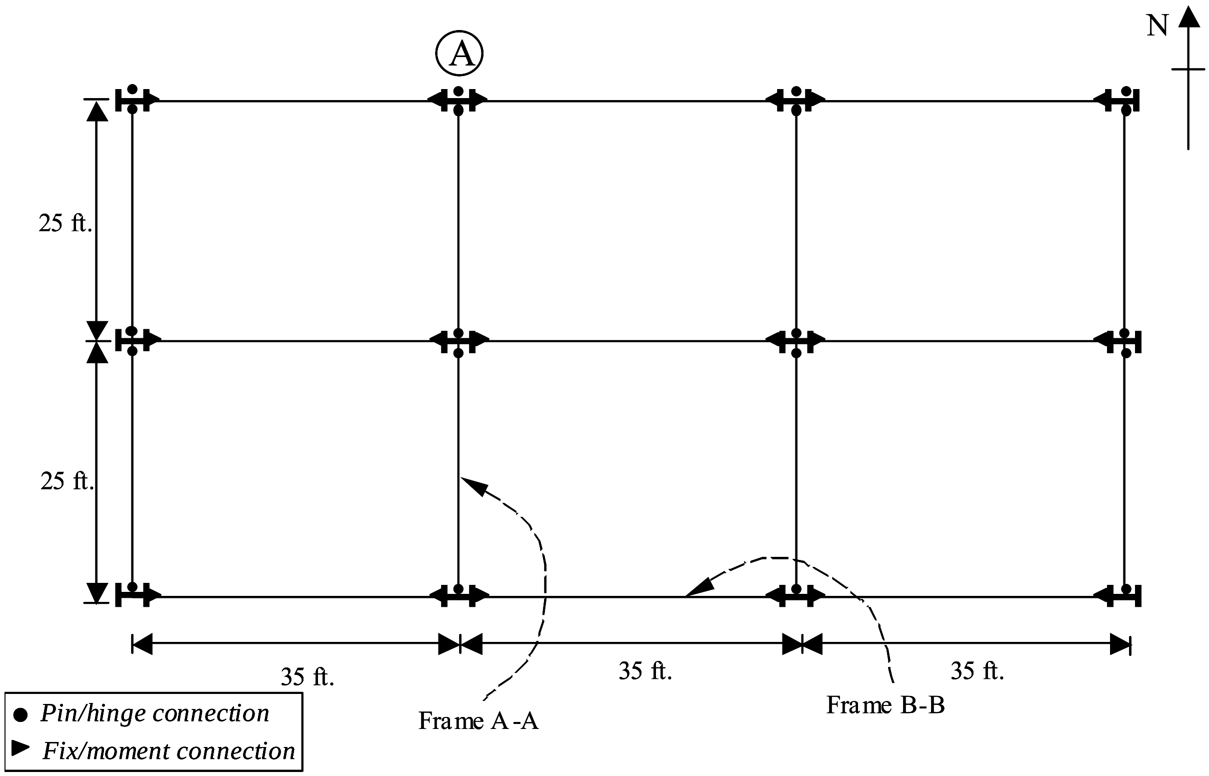


Figure 1. Structural floor plan and layout

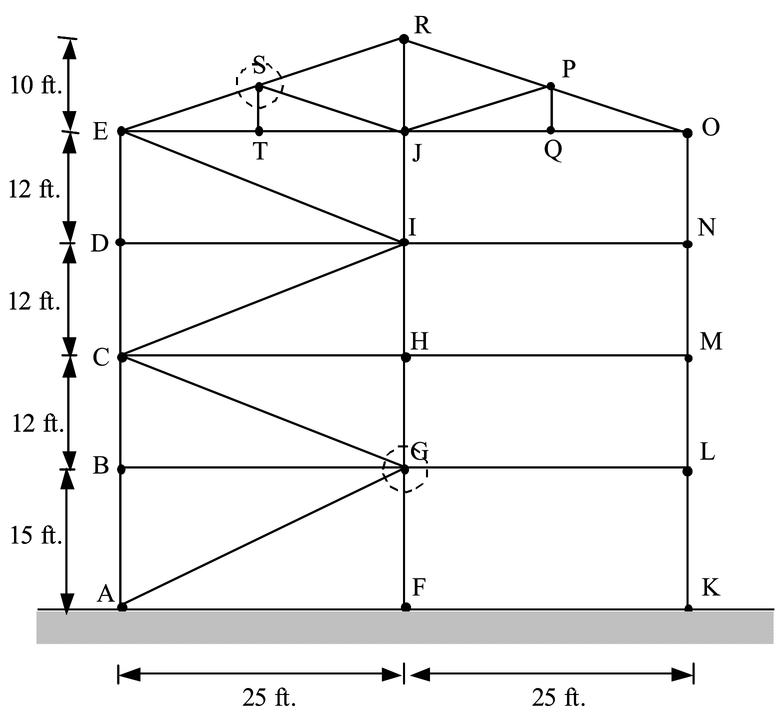


Figure 2. Structural elevation of frame A-A

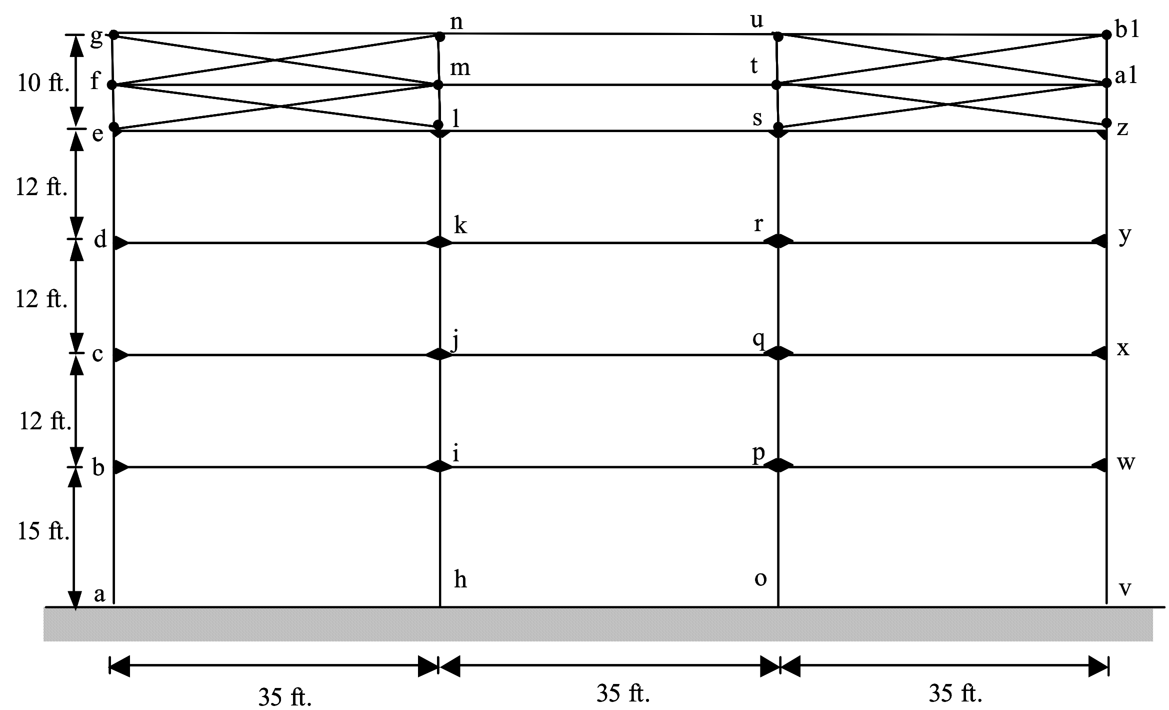
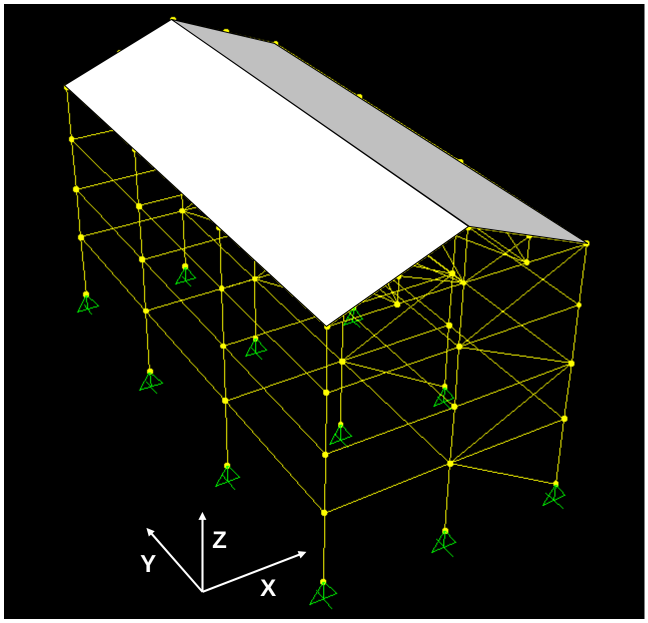


Figure 3. Structural elevation of frame B-B

- As shown in Figure 1, the building has two 25-ft. bays in the *north-south* direction and three 35 ft. bays in the *east-west* direction.
- There are *four* structural frames in the north-south direction. These frames have structural elevations similar to frame A-A shown in Figure 2.
- There are *three* structural frames in the east-west directions. These frames have structural elevations similar to frame B-B shown in Figure 3.
- The building has a *roof truss*, which is shown in Figures 2 and 3.
- Frame A-A is a braced frame, where all members are connected using *pin/hinge connections*. Diagonal bracing members are needed for stability.
- Frame B-B is a moment frame, where all members are connected using *fix/moment connections*. There is no need for diagonal bracing members.
- The north-south and east-west frames resist the *vertical gravity* loads together.
- The three moment frames in the east-west direction resist the *horizontal lateral loads* in the east-west direction.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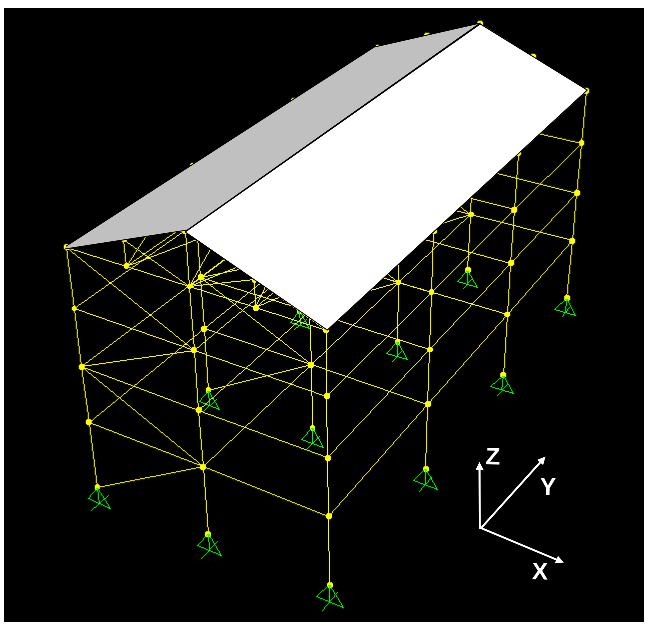
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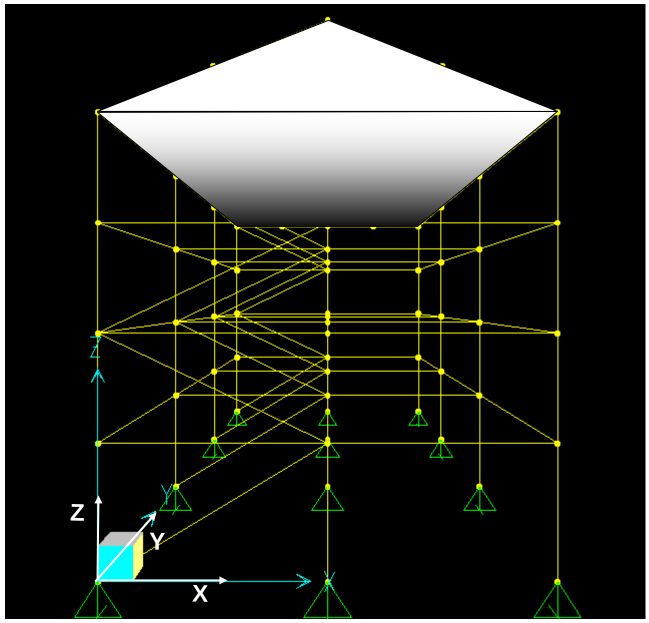
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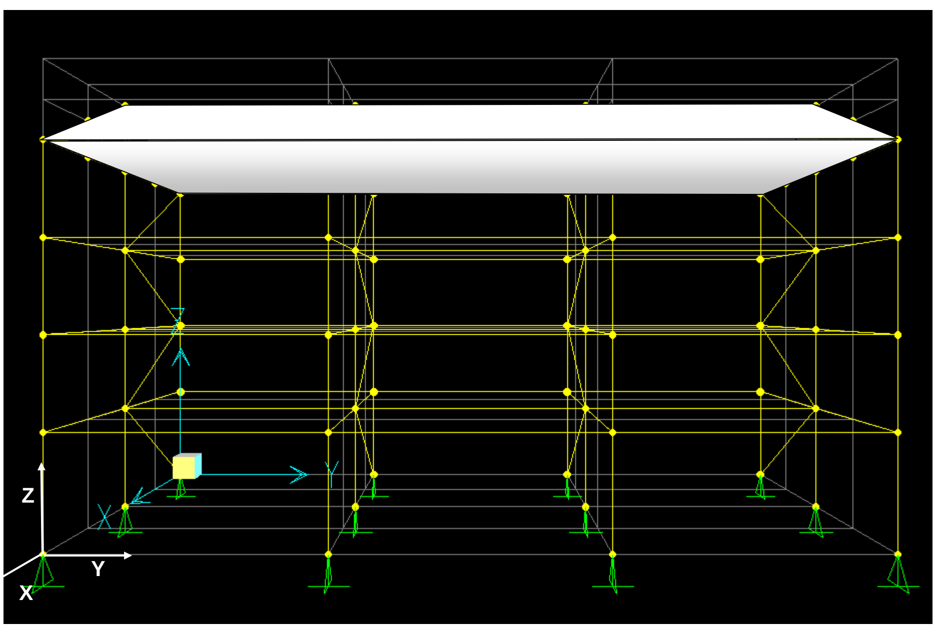
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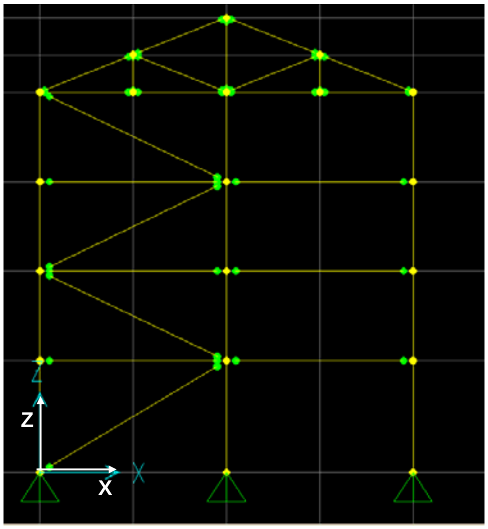
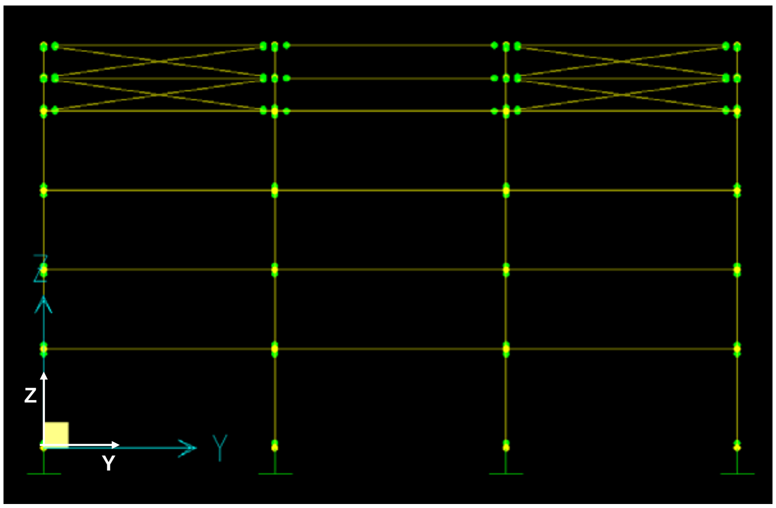
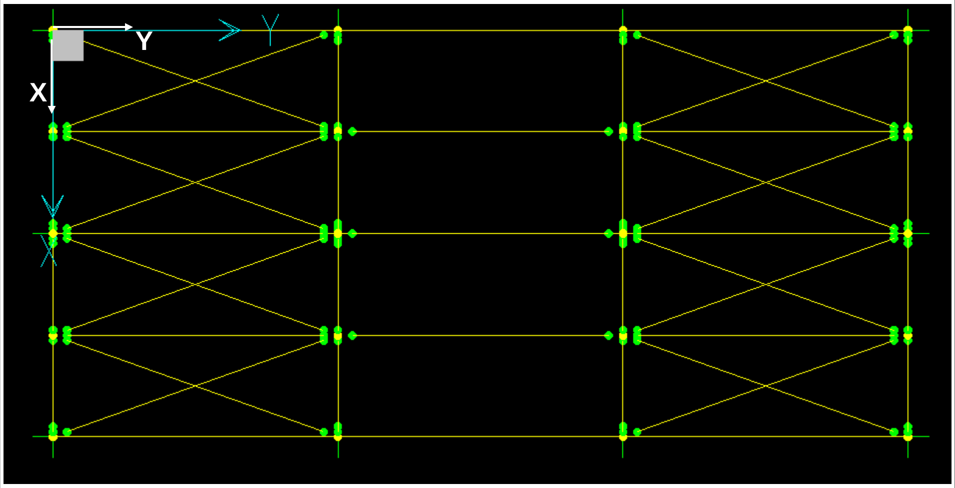


ELEVATION OF BRACED FRAME

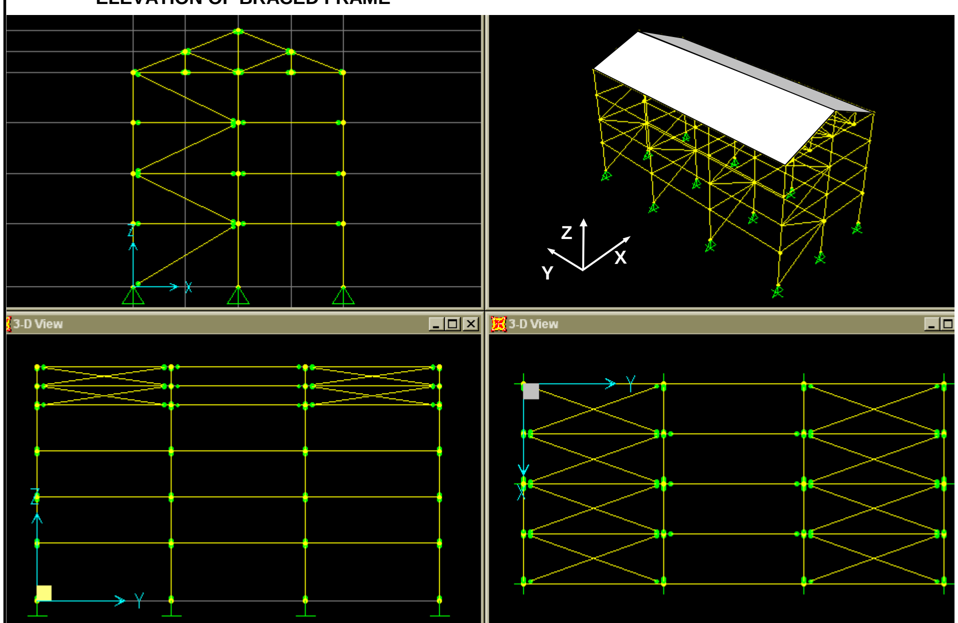


ELEVATION OF MOMENT FRAME



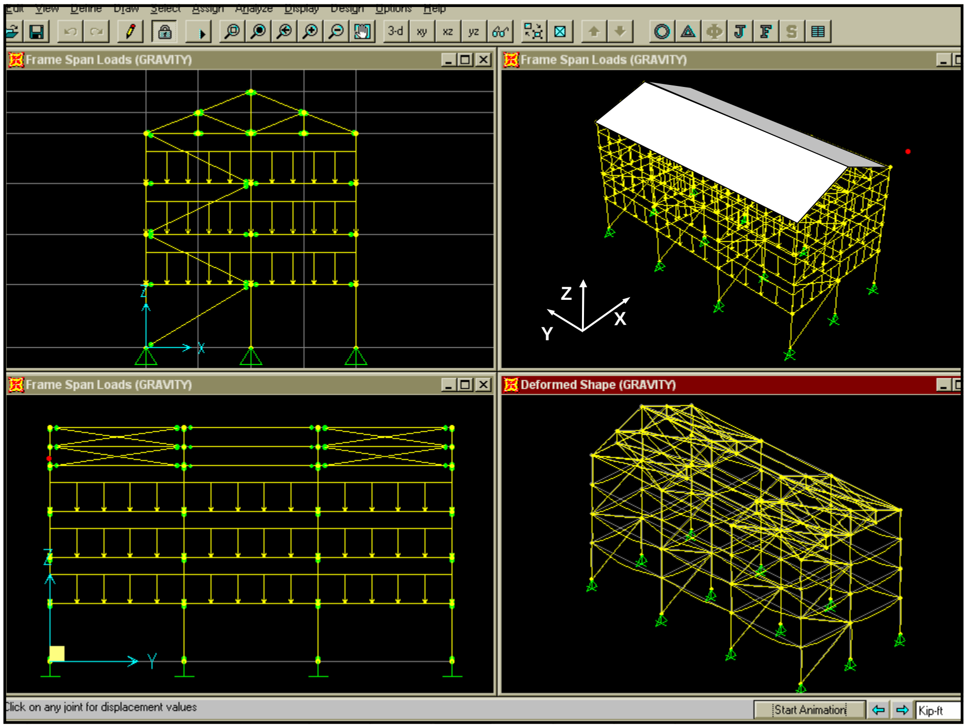
PLAN OF ROOF TRUSS

ELEVATION OF BRACED FRAME



ELEVATION OF MOMENT FRAME

PLAN OF ROOF TRUSS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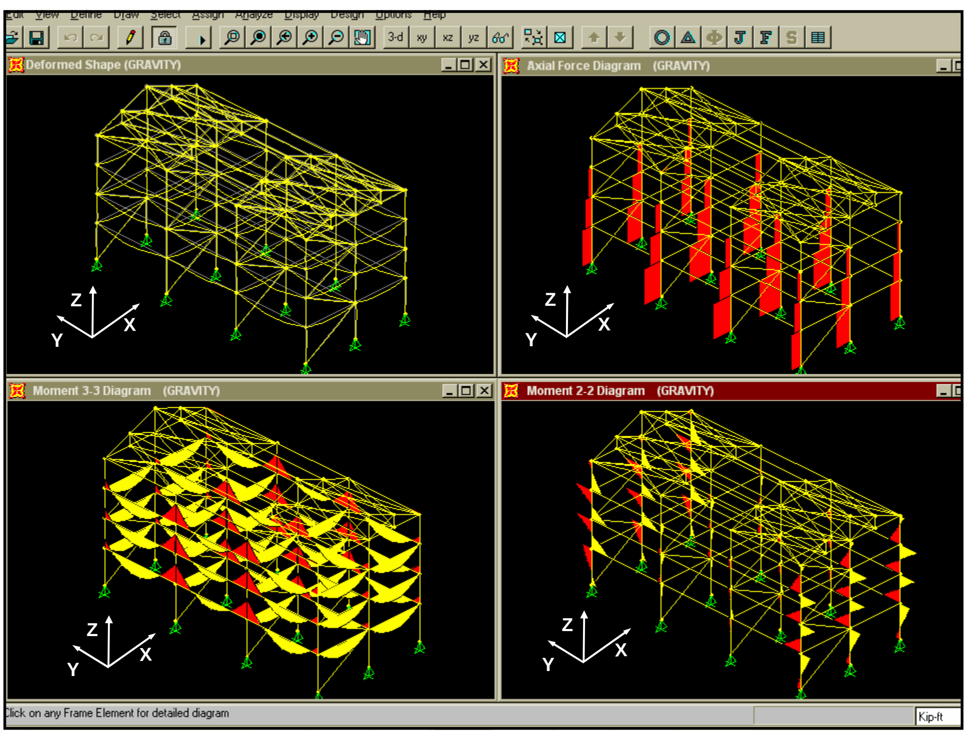
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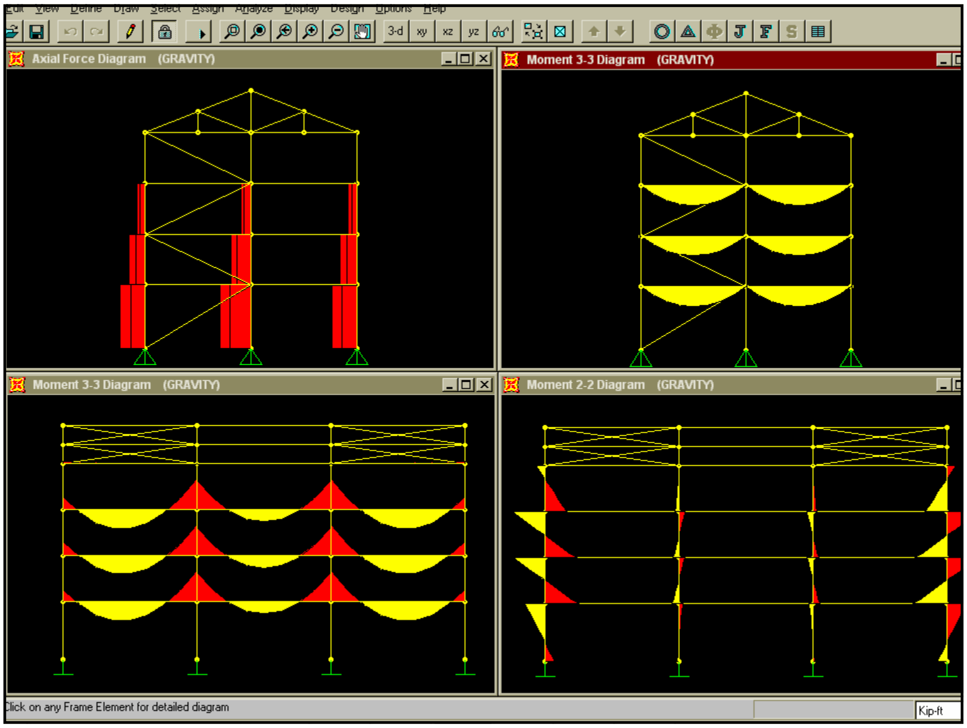
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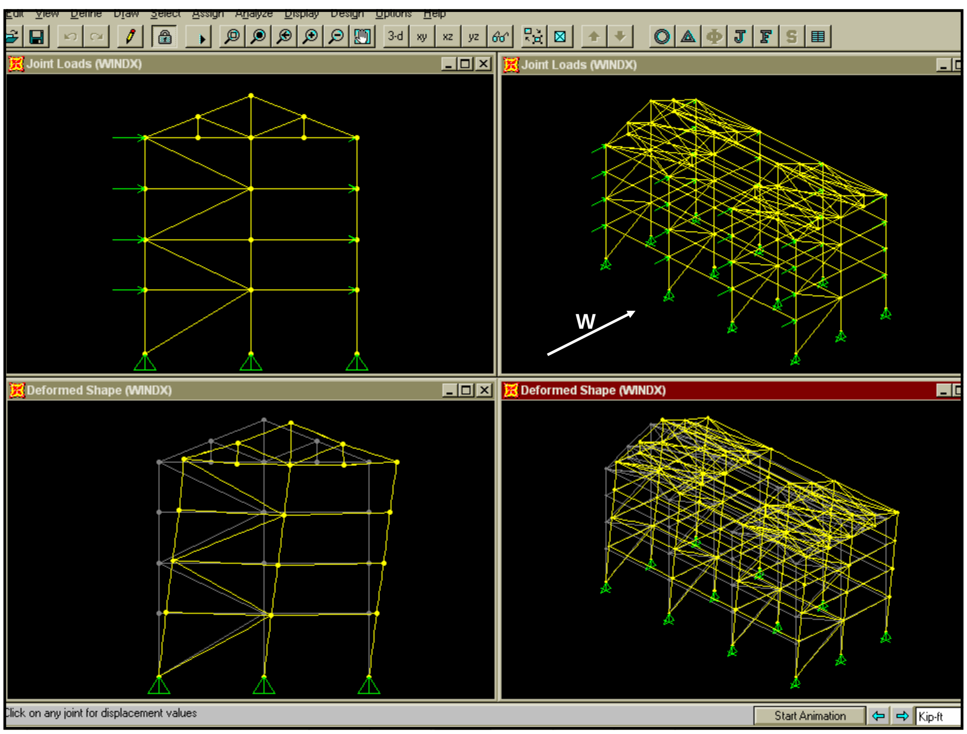
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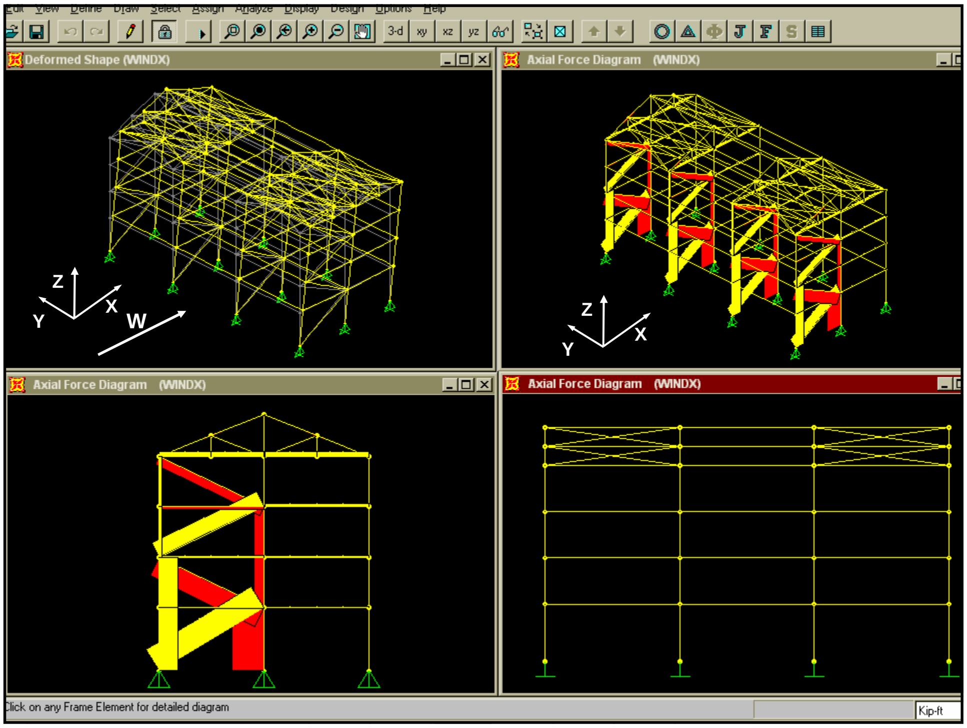
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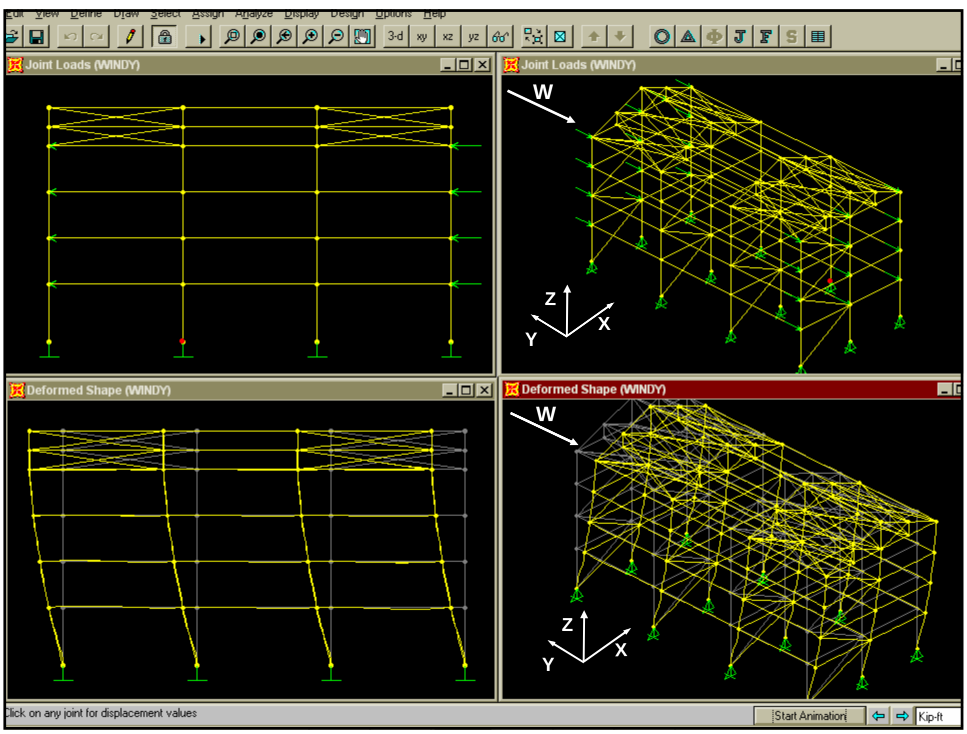
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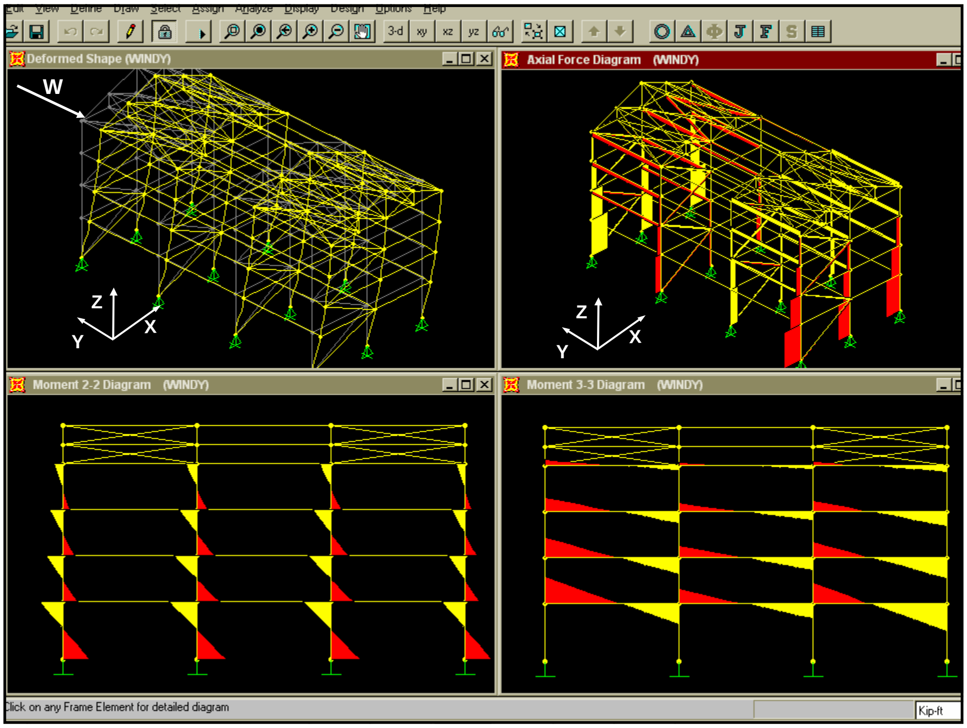
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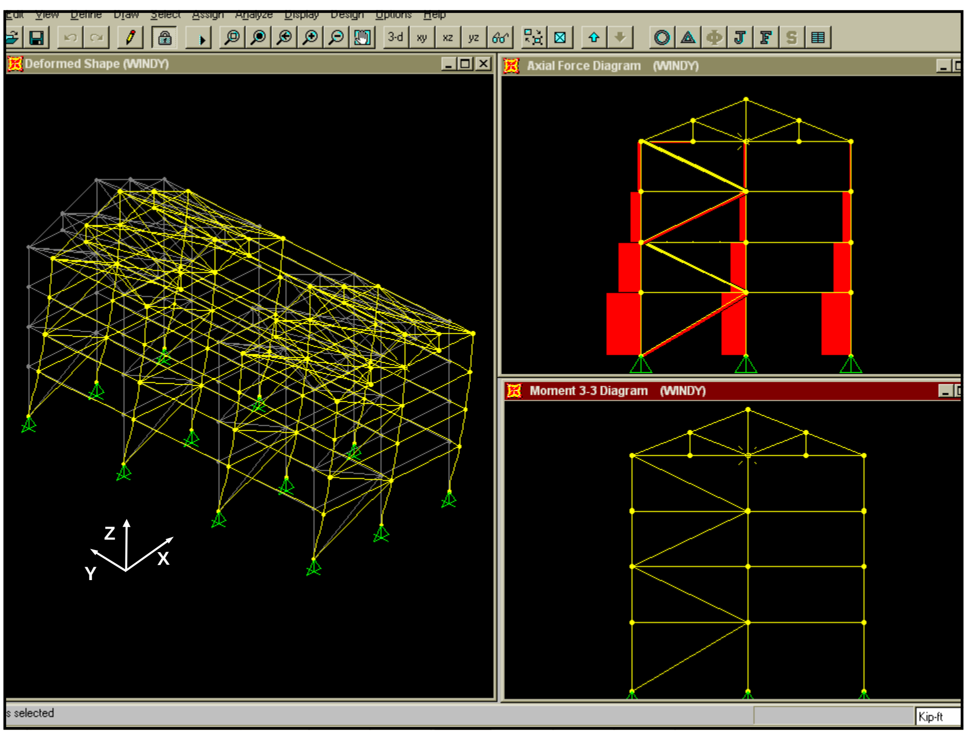
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- The four braced frames in the north-south direction resist the *horizontal lateral loads* in the north-south direction.

1.4 STRUCTURAL MEMBERS

Structural members are categorized based up on the internal forces in them. For example:

- <u>Tension member</u> –subjected to tensile axial force only
- <u>Column or compression member</u> –subjected to compressive axial force only
- <u>Tension/Compression member</u> –subjected to tensile/compressive axial forces
- <u>Beam member</u> –subjected to flexural loads, i.e., shear force and bending moment only. The axial force in a beam member is negligible.
- <u>Beam-column member</u> – member subjected to combined axial force and flexural loads (shear force, and bending moments)

In basic structural analysis ***(CE305)*** students have come across two types of structures, namely, *trusses and frames*. For example, Figure 2 shows a roof truss supported by a braced frame.

- All the members of a truss are connected using pin/hinge connections. All external forces are applied at the pins/hinges. As a result, all truss members are subjected to axial forces (tension or compression) only.
- In braced and moment frames, the horizontal members (beams) are subjected to flexural loads only.
- In braced frames, the vertical members (columns) are subjected to compressive axial forces only.
- In braced frames, the diagonal members (braces) are subjected to tension/compression axial forces only.
- In moment frames, the vertical members (beam-columns) are subjected to combined axial and flexural loads.

For practice, let us categorize the member shown in Figures 2 and 3.

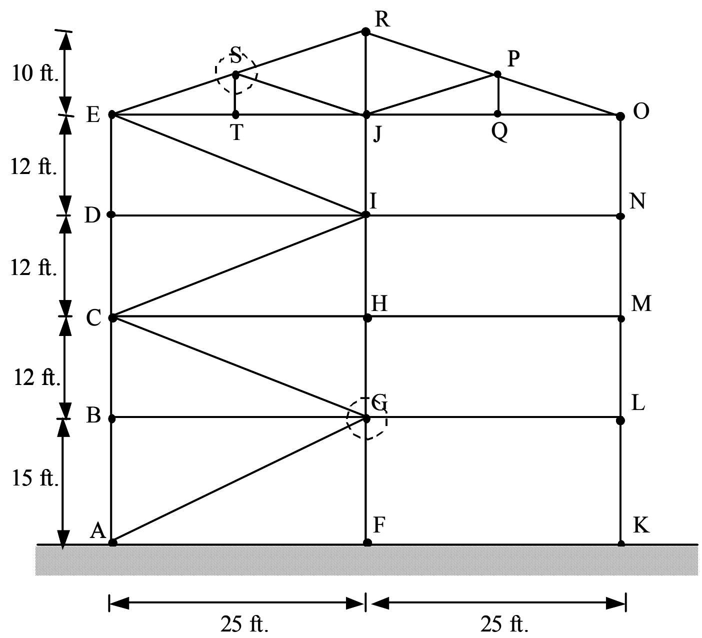


Figure 2. Structural elevation of frame A-A

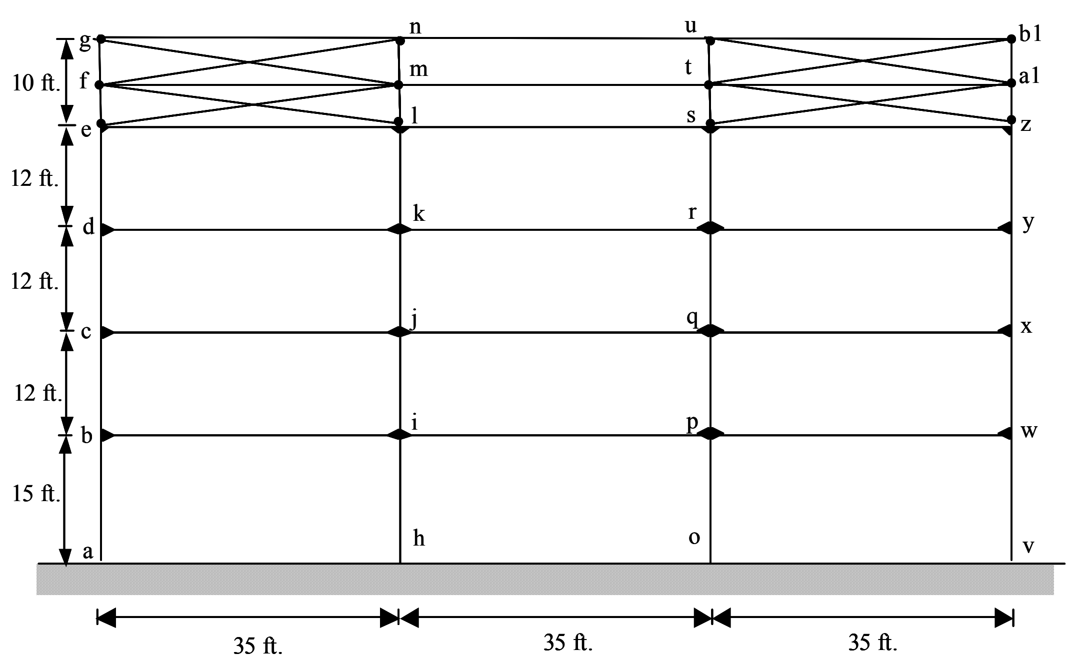


Figure 3. Structural elevation of frame B-B

1.5 STRUCTURAL CONNECTIONS

Members of a structural frame are connected together using connections. Prominent connection types include: (1) truss / bracing member connections; (2) simple shear connections; (3) fully-restrained moment connections; and (4) partially-restrained flexible moment connections.

- Truss / bracing member connections are used to connect two or more truss members together. Only the *axial forces* in the members have to be transferred through the connection for continuity.
- Simple shear connections are the *pin connections* used to connect beam to column members. Only the *shear forces* are transferred through the connection for continuity. The *bending moments* are not transferred through the connection.
- Moment connections are *fix connections* used to connect beam to column members. Both the shear forces and bending moments are transferred through the connections with very small deformations (*full restraint*).
- Partially restrained connections are *flexible connections* used to connect beam to column members. The shear forces are transferred fully through the connection. However, the bending moment is only transferred partially.

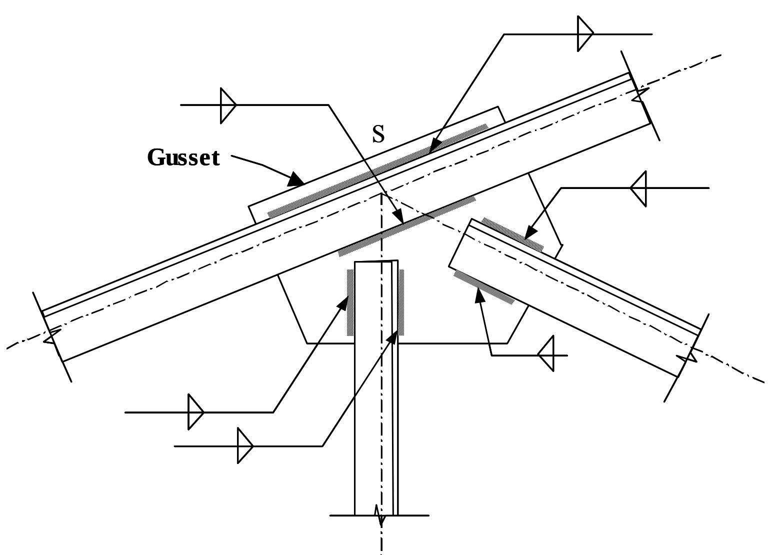


Figure 4. Truss connection at **S** in Frame A-A.

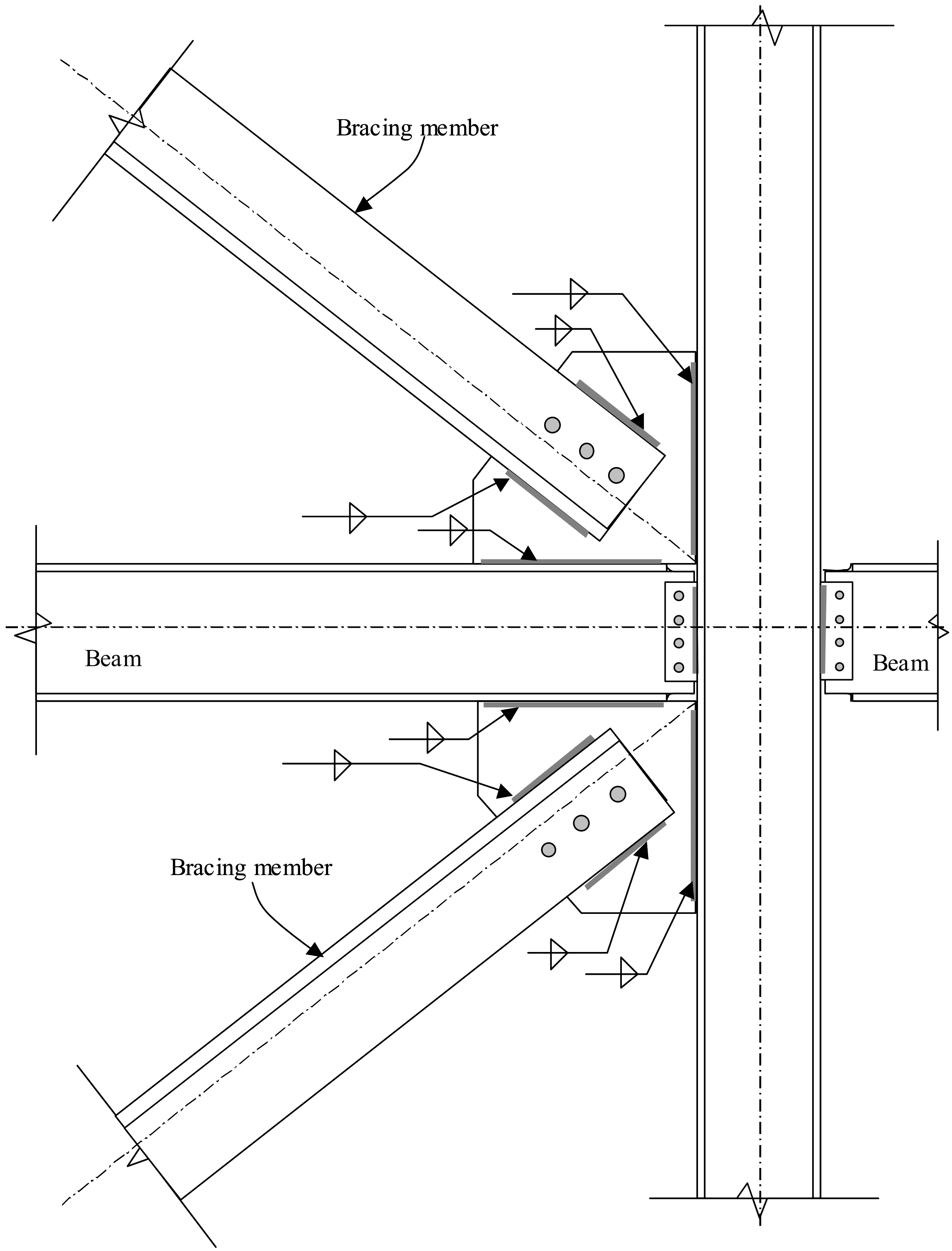


Figure 5. Bracing connection and Simple Shear Connection at **G** in Frame A-A.

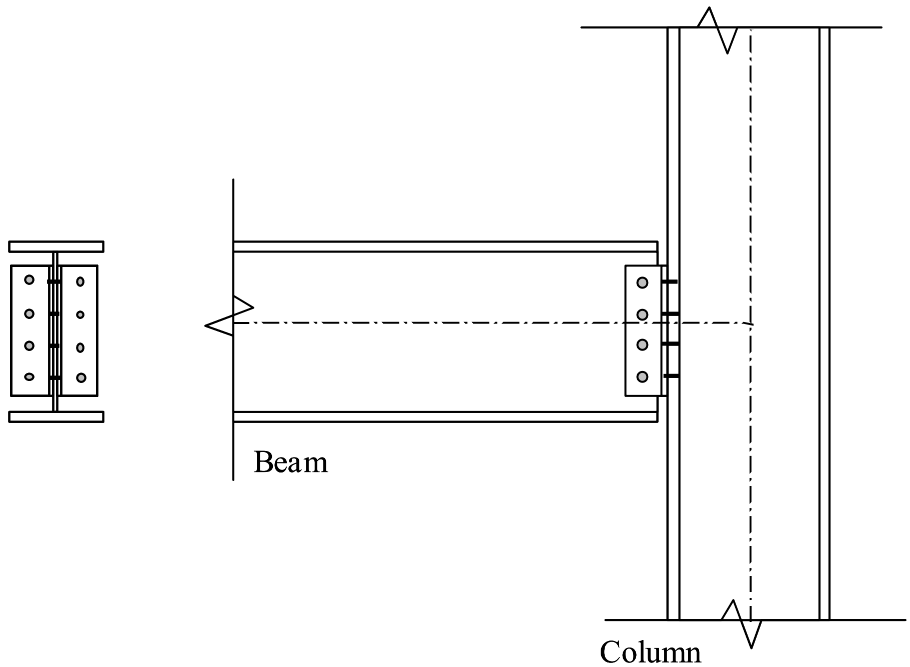


Figure 6. All-bolted double angle shear connection.

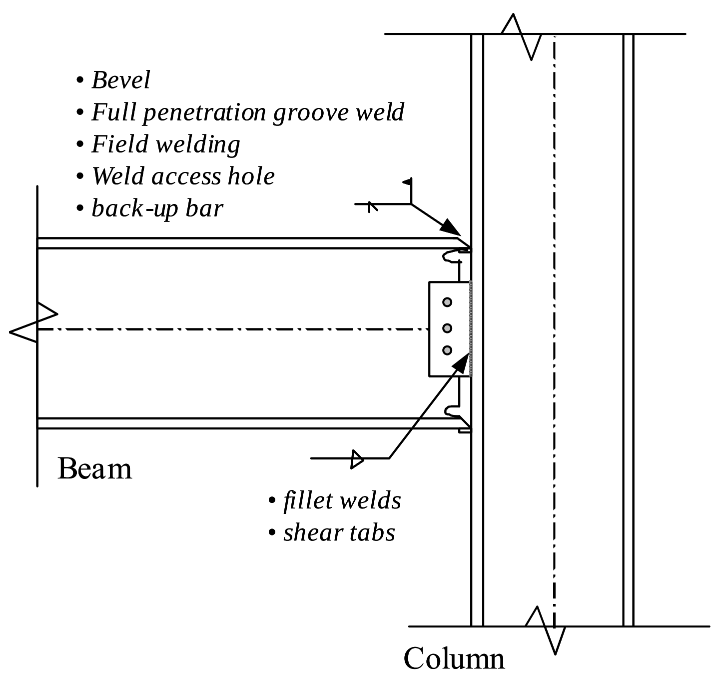


Figure 7. Directly welded flange fully restrained moment connection.

- Figure 4 shows an example truss connection. Figure 5 shows an example bracing connection. Figure 6 shows an example shear connection. Figure 7 shows an example moment connection.
- Connections are developed using bolts or welds.
- Bolts are used to connect two or more plate elements that are in the same plane. Bolt-holes are drilled in the plate elements. The *threaded* bolt shank passes through the holes, and the connection is secured using *nuts*.
- Bolts are usually made of *higher strength steel*.
- Welds can be used to connect plate elements that are in the same or different planes. A high voltage *electric arc* is developed between the two plate elements. The electric arc causes localized melting of the base metal (plate element) and the weld electrode. After cooling, all the molten metal (base and weld) solidifies into one *continuum*. Thus, developing a welded connection.
- In Figure 4, all the truss members are connected together by welding to a common *gusset* plate. The axial forces in the members are transferred through the *gusset* plates. This same connection can also be developed using bolts. *How?*
- In Figure 5, the bracing members are connected to *gusset* plates, which are also connected to the beam and column. The bracing member can be connected to the *gusset* plate using bolts or welds. However, the *gusset* plate has to be welded to the beam / column.
- In Figure 6, two angles are bolted to the web of the beam. The perpendicular legs of the angles are bolted to the flange of the column. Thus, an all-bolted double-angle shear connection is achieved. This all-bolted connection will be easier to assemble in the field as compared to welding. *How is this a shear connection?*
- In Figure 7, the beam flanges are *beveled* and welded directly to the flange of column using full penetration groove welds. This welding will have to be done in the *field* during erection and it will require the use of back-up bars. Weld-access holes and skilled welders are required to achieve a weld of acceptable quality.
- In Figure 7, the beam web is bolted to a shear tab (plate), which is fillet welded to the column in the shop. This shear tab connection transfers the shear from the beam to the column. *How is Figure 7 a moment connection?*

1.6 Structural Loads

The building structure must be designed to carry or resist the loads that are applied to it over its design-life. The building structure will be subjected to loads that have been categorized as follows:

- Dead Loads (*D*): are permanent loads acting on the structure. These include the self-weight of structural and non-structural components. They are usually *gravity* loads.
- Live Loads (*L*): are non-permanent loads acting on the structure due to its use and occupancy. The magnitude and location of live loads changes frequently over the design life. Hence, they cannot be estimated with the same accuracy as dead loads.
- Wind Loads (*W*): are in the form of *pressure* or *suction* on the exterior surfaces of the building. They cause horizontal lateral loads (forces) on the structure, which can be critical for tall buildings. Wind loads also cause *uplift* of light roof systems.
- Snow Loads (*S*): are vertical gravity loads due to snow, which are subjected to variability due to seasons and drift.
- Roof Live Load (L_r): are live loads on the roof caused during the design life by planters, people, or by workers, equipment, and materials during maintenance.
- Values of structural loads are given in the publication ASCE 7-98: *Minimum Design Loads for Buildings and Other Structures.* The first phase of structural design consists of estimating the loads acting on the structure. This is done using the load values and combinations presented in ASCE 7-98 as explained in the following sub-sections.

1.6.1 Step I. Categorization of Buildings

- Categories I, II, III, and IV. See Table 1.1 below and in ASCE 7-98.

TABLE 1-1. Classification of Buildings and Other Structures for Flood, Wind, Snow, and Earthquake Loads

Nature of Occupancy	Category
Buildings and other structures that represent a low hazard to human life in the event of failure including, but not limited to: • Agricultural facilities • Certain temporary facilities • Minor storage facilities	I
All buildings and other structures except those listed in Categories I, III and IV	II
Buildings and other structures that represent a substantial hazard to human life in the event of failure including, but not limited to: • Buildings and other structures where more than 300 people congregate in one area • Buildings and other structures with day-care facilities with capacity greater than 150 • Buildings and other structures with elementary or secondary school facilities with capacity greater than 150 • Buildings and other structures with a capacity greater than 500 for colleges or adult education facilities • Health care facilities with a capacity of 50 or more resident patients but not having surgery or emergency treatment facilities • Jails and detention facilities • Power generating stations and other public utility facilities not included in Category IV Buildings and other structures containing sufficient quantities of toxic, explosive or other hazardous substances to be dangerous to the public if released including, but not limited to: • Petrochemical facilities • Fuel storage facilities • Manufacturing or storage facilities for hazardous chemicals • Manufacturing or storage facilities for explosives Buildings and other structures that are equipped with secondary containment of toxic, explosive or other hazardous substances (including, but not limited to double wall tank, dike of sufficient size to contain a spill, or other means to contain a spill or a blast within the property boundary of the facility and prevent release of harmful quantities of contaminants to the air, soil, ground water, or surface water) or atmosphere (where appropriate) shall be eligible for classification as a Category II structure. In hurricane prone regions, buildings and other structures that contain toxic, explosive, or other hazardous substances and do not qualify as Category IV structures shall be eligible for classification as Category II structures for wind loads if these structures are operated in accordance with mandatory procedures that are acceptable to the authority having jurisdiction and which effectively diminish the effects of wind on critical structural elements or which alternatively protect against harmful releases during and after hurricanes.	III
Buildings and other structures designated as essential facilities including, but not limited to: • Hospitals and other health care facilities having surgery or emergency treatment facilities • Fire, rescue and police stations and emergency vehicle garages • Designated earthquake, hurricane, or other emergency shelters • Communications centers and other facilities required for emergency response • Power generating stations and other public utility facilities required in an emergency • Ancillary structures (including, but not limited to communication towers, fuel storage tanks, cooling towers, electrical substation structures, fire water storage tanks or other structures housing or supporting water or other fire-suppression material or equipment) required for operation of Category IV structures during an emergency • Aviation control towers, air traffic control centers and emergency aircraft hangars • Water storage facilities and pump structures required to maintain water pressure for fire suppression • Buildings and other structures having critical national defense functions	IV

1.6.2 Dead Loads (D)

Dead loads consist of the weight of all materials of construction incorporated into the building including but not limited to walls, floors, roofs, ceilings, stairways, built-in partitions, finishes, cladding and other similarly incorporated architectural and structural items, and fixed service equipment such as plumbing stacks and risers, electrical feeders, and heating, ventilating, and air conditioning systems.

In some cases, the structural dead load can be estimated satisfactorily from simple formulas based in the weights and sizes of similar structures. For example, the average weight of steel framed buildings is 60-75 lb/ft^2, and the average weight for reinforced concrete buildings is 110 - 130 lb/ft^2.

From an engineering standpoint, once the materials and sizes of the various components of the structure are determined, their weights can be found from tables that list their densities. See Tables 1.2 and 1.3, which are taken from Hibbeler, R.C. (1999), *Structural Analysis,* 4th Edition.

Table 1–2 Minimum Densities for Design Loads from Materials*

	lb/ft^3	kN/m^3
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

Table 1–3 Minimum Design Dead Loads*

Walls	psf	kN/m^2
4-in. (102 mm) clay brick	39	1.87
8-in. (203 mm) clay brick	79	3.78
12-in. (305 mm) clay brick	115	5.51
Frame Partitions and Walls		
Exterior stud walls with brick veneer	48	2.30
Windows, glass, frame and sash	8	0.38
Wood studs 2 × 4, (51 × 102) unplastered	4	0.19
Wood studs 2 × 4, (51 × 102) plastered one side	12	0.57
Wood studs 2 × 4, (51 × 102) plastered two sides	20	0.96
Floor Fill		
Cinder concrete, per inch (mm)	9	0.017
Lightweight concrete, plain, per inch (mm)	8	0.015
Stone concrete, per inch (mm)	12	0.023
Ceilings		
Acoustical fiberboard	1	0.05
Plaster on tile or concrete	5	0.24
Suspended metal lath and gypsum plaster	10	0.48
Asphalt shingles	2	0.10
Fiberboard, $\frac{1}{2}$-in. (13 mm)	0.75	0.04

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures,* ANSI/ASCE 7-95. Copies of this standard may be purchased from ASCE at 345 East 47th Street, New York, N.Y. 10017-2398.

1.6.3 Live Loads

- Building floors are usually subjected to uniform live loads or concentrated live loads. They have to be designed to safely support the *minimum uniformly distributed load* or the *minimum concentrated live load* values given in the ASCE 7-98 (see Table 1.4 below), whichever produces the maximum load effects in the structural members.

TABLE 4-1. Minimum Uniformly Distributed Live Loads, L_o and Minimum Concentrated Live Loads

Occupancy or Use	Uniform psf (kN/m^2)	Concentration lb (kN)
Apartments (see residential)		
Access floor systems		
Office use	50 (2.4)	2,000 (8.9)
Computer use	100 (4.79)	2,000 (8.9)
Armories and drill rooms	150 (7.18)	
Assembly areas and theaters		
Fixed seats (fastened to floor)	60 (2.87)	
Lobbies	100 (4.79)	
Movable seats	100 (4.79)	
Platforms (assembly)	100 (4.79)	
Stage floors	150 (7.18)	
Balconies (exterior)	100 (4.79)	
On one- and two-family residences only, and not exceeding 100 ft^2 (9.3 m^2)	60 (2.87)	
Bowling alleys, poolrooms and similar recreational areas	75 (3.59)	
Catwalks for maintenance access	40 (1.92)	300 (1.33)
Corridors		
First floor	100 (4.79)	
Other floors, same as occupancy served except as indicated		
Dance halls and ballrooms	100 (4.79)	
Decks (patio and roof)		
Same as area served, or for the type of occupancy accommodated		
Dining rooms and restaurants	100 (4.79)	
Dwellings (see residential)		
Elevator machine room grating [on area of 4 in.^{2} (2,580 mm^2)]		300 (1.33)
Finish light floor plate construction [on area of 1 in.^{2} (645 mm^2)]		200 (0.89)
Fire escapes	100 (4.79)	
On single-family dwellings only	40 (1.92)	
Fixed Ladders		See Section 4.4
Garages (passenger cars only)	50 (2.40)	—[1]
Trucks and buses		—[2]
Grandstands (see stadium and arena bleachers)		
Gymnasiums, main floors and balconies	100 (4.79)[4]	
Handrails, guardrails and grab bars	See Section 4.4	
Hospitals		
Operating rooms, laboratories	60 (2.87)	1,000 (4.45)
Private rooms	40 (1.92)	1,000 (4.45)
Wards	40 (1.92)	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
Hotels (see residential)		
Libraries		
Reading rooms	60 (2.87)	1,000 (4.45)
Stack rooms	150 (7.18)[3]	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
Manufacturing		
Light	125 (6.00)	2,000 (8.90)
Heavy	250 (11.97)	3,000 (13.40)
Marquees and Canopies	75 (3.59)	
Office Buildings		
File and computer rooms shall be designed for heavier loads based on anticipated occupancy		
Lobbies and first floor corridors	100 (4.79)	2,000 (8.90)
Offices	50 (2.40)	2,000 (8.90)
Corridors above first floor	80 (3.83)	2,000 (8.90)

TABLE 4-1. Minimum Uniformly Distributed Live Loads, L_o and Minimum Concentrated Live Loads (*Continued*)

Occupancy or Use	Uniform psf (kN/m^2)	Concentration lb (kN)
Penal Institutions		
Cell blocks	40 (1.92)	
Corridors	100 (4.79)	
Residential		
Dwellings (one- and two-family)		
Uninhabitable attics without storage	10 (0.48)	
Uninhabitable attics with storage	20 (0.96)	
Habitable attics and sleeping areas	30 (1.44)	
All other areas except stairs and balconies	40 (1.92)	
Hotels and multifamily houses		
Private rooms and corridors serving them	40 (1.92)	
Public rooms and corridors serving them	100 (4.79)	
Reviewing stands, grandstands and bleachers	100 (4.79)[4]	
Roofs	See Sections 4.3 and 4.9	
Schools		
Classrooms	40 (1.92)	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
First floor corridors	100 (4.79)	1,000 (4.45)
Scuttles, skylight ribs, and accessible ceilings		200 (9.58)
Sidewalks, vehicular driveways, and yards, subject to trucking	250 (11.97)[5]	8,000 (35.60)[6]
Stadiums and Arenas		
Bleachers	100 (4.79)[4]	
Fixed Seats (fastened to floor)	60 (2.87)[4]	
Stairs and exitways	100 (4.79)	—[7]
One- and two-family residences only	40 (1.92)	
Storage areas above ceilings	20 (0.96)	
Storage warehouses (shall be designed for heavier loads if required for anticipated storage)		
Light	125 (6.00)	
Heavy	250 (11.97)	
Stores		
Retail		
First floor	100 (4.79)	1,000 (4.45)
Upper floors	73 (3.59)	1,000 (4.45)
Wholesale, all floors	125 (6.00)	1,000 (4.45)
Vehicle barriers	See Section 4.4	
Walkways and elevated platforms (other than exitways)	60 (2.87)	
Yards and terraces, pedestrians	100 (4.79)	

[1]Floors in garages or portions of building used for the storage of motor vehicles shall be designed for the uniformly distributed live loads of Table 4-1 or the following concentrated load: (1) for passenger cars accommodating not more than nine passengers, 2,000 lb (8.90 kN) acting on an area of 20 in.^{2} (12,900 mm^2); (2) mechanical parking structures without slab or deck, passenger car only, 1,500 lb (6.70 kN) per wheel.

[2]Garages accommodating trucks and buses shall be designed in accordance with an approved method which contains provisions for truck and bus loadings.

[3]The weight of books and shelving shall be computed using an assumed density of 65 lb/ft^3 (pounds per cubic foot, sometimes abbreviated pcf) (10.21 kN/m^3) and converted to a uniformly distributed load; this load shall be used if it exceeds 150 lb/ft^2 (7.18 kN/m^2).

[4]In addition to the vertical live loads, horizontal swaying forces parallel and normal to the length of seats shall be included in the design according to the requirements of ANSI/NFPA 102 [3].

[5]Other uniform loads in accordance with an approved method which contains provisions for truck loadings shall also be considered where appropriate.

[6]The concentrated wheel load shall be applied on an area of 20 in.^{2} (12,900 mm^2).

[7]Minimum concentrated load on stair treads [on area of 4 in.^{2} (2,580 mm^2)] is 300 lb (1.33 kN).

- The minimum uniformly distributed live loads (L_o) given in Table 1.4 above can be reduced for buildings with *very large floor areas*, because it is unlikely that the prescribed live load will occur simultaneously throughout the entire structure.

- Equation (1.1) can be used to calculate the reduce uniformly distributed live load (L)

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{K_{LL} A_T}} \right) \tag{1.1}$$

where, A_T is the tributary area in ft^2 and K_{LL} is the live load element factor as follows:

K_{LL} is equal to 4.0 for interior columns and exterior columns without cantilever slabs. K_{LL} is equal to 3.0 for edge columns with cantilever slabs.

K_{LL} is equal to 2.0 for corner columns with cantilever slabs, edge beams without cantilever slabs, and interior beams.

K_{LL} is equal to 1.0 for all other members not identified above.

- Some limitations to the live load reduction are as follows:

L cannot be less than $0.5L_o$ for members supporting one floor and L cannot be less that $0.4L_o$ for members supporting two or more floors.

Live loads that exceed 100 lb/ft^2 shall not be reduced except the live loads for members supporting two or more floors may be reduced by 20%.

Live loads exceeding 100 lb/ft^2 shall not be reduced for passenger car garages, public assembly occupancies, or roofs

1.6.4 Roof Live Loads

Ordinary flat, pitched, and curved roofs shall be designed for the live loads specified in Equation 1.2 (from ASCE 7-98).

$$L_r = 20\ R_1\ R_2 \qquad \text{where, } 12 \leq L_r \leq 20 \qquad (1.2)$$

where,

L_r is the roof live load per square foot of horizontal projection in psf.

	$= 1$	for $A_T \leq 200\ ft^2$
R_1	$= 1.2 - 0.001\ A_T$	for $200 < A_T < 600\ ft^2$
	$= 0.6$	for $600 ft^2 \leq A_T$

	$= 1$	for $F \leq 4$
R_2	$= 1.2 - 0.05\ F$	for $4 < F < 12$
	$= 0.6$	for $12 \leq F$

where, F = no. of inches of rise per foot for pitched roof.

1.6.5 Wind Loads

- Design wind loads for buildings can be based on: (a) simplified procedure; (b) analytical procedure; and (c) wind tunnel or small-scale procedure.

- Refer to ASCE 7-98 for the simplified procedure. This simplified procedure is applicable only to buildings with mean roof height less than 30 ft.

- The wind tunnel procedure consists of developing a small-scale model of the building and testing it in a wind tunnel to determine the expected wind pressures etc. It is expensive and may be utilized for difficult or special situations.

- The analytical procedure is used in most design offices. It is fairly systematic but somewhat complicated to account for the various situations that can occur:

- Wind velocity will cause pressure on any surface in its path. The wind velocity and hence the velocity pressure depend on the height from the ground level. Equation 1.3 is recommended by ASCE 7-98 for calculating the velocity pressure (q_z) in lb/ft^2

$$q_z = 0.00256\, K_z\, K_{zt}\, K_d\, V^2\, I \quad (\text{lb/ft}^2) \tag{1.3}$$

where, V is the wind velocity (see Figure 6-1 in ASCE 7-98)

K_d is a directionality factor (=0.85 for CE 405)

K_{zt} is a topographic factor (= 1.0 for CE 405)

I is the importance factor (=1.0 for CE 405)

K_z varies with height z above the ground level (see Table 6-5 in ASCE 7-98)

- A significant portion of the U.S. including Lansing has V = 90 mph. At these location

$$q_z = 17.625\, K_z \quad (\text{lb/ft}^2) \tag{1.4}$$

- The velocity pressure q_z is used to calculate the design wind pressure (p) for the building structure as follows:

$$p = q\,GC_p - q_i\,(GC_{pi}) \qquad (\text{lb/ft}^2) \qquad (1.5)$$

where, G = gust effect factor (=0.85 for CE 405)

C_p = external pressure coefficient from Figure 6-3 in ASCE 7-98

C_{pi} = internal pressure coefficient from Table 6-7 in ASCE 7-98

q depends on the orientation of the building wall or roof with respect to direction of the wind as follows:

$q = q_z$ for the windward wall – varies with height z

$q = q_h$ for leeward wall.
q_h is q_z evaluated at $z = h$ (mean height of building). q_h is constant.

$q_i = q_h$ for windward, leeward, side walls and roofs.

- Note that a positive sign indicates pressure acting towards a surface. Negative sign indicate pressure away from the surface
- Equation 1.5 indicates that the design wind pressure p consists of two components: (1) the external pressure on the building ($q\,GC_p$); and (2) the internal pressure in the building ($q_h\,GC_{pi}$)

1.6.6 Load and Resistance Factor Design

The load and resistance factor design approach is recommended by AISC for designing steel structures. It can be understood as follows:

Step I. Determine the ultimate loads acting on the structure

- The values of D, L, W, etc. given by ASCE 7-98 are nominal loads (not maximum or ultimate)

- During its design life, a structure can be subjected to some maximum or ultimate loads caused by combinations of D, L, or W loading.
- The ultimate load on the structure can be calculated using factored load combinations, which are given by ASCE and AISC (see pages 2-10 and 2-11 of AISC manual). The most relevant of these load combinations are given below:

$$1.4\ D \qquad (4.2-1)$$

$$1.2\ D + 1.6\ L + 0.5\ (L_r \text{ or } S) \qquad (4.2-2)$$

$$1.2\ D + 1.6\ (L_r \text{ or } S) + (0.5\ L \text{ or } 0.8\ W) \qquad (4.2-3)$$

$$1.2\ D + 1.6\ W + 0.5\ L + 0.5\ (L_r \text{ or } S) \qquad (4.2-4)$$

$$0.9\ D + 1.6\ W \qquad (4.2-5)$$

Step II. Conduct linear elastic structural analysis

- Determine the design forces (P_u, V_u, and M_u) for each structural member

Step III. Design the members

- The failure (design) strength of the designed member must be greater than the corresponding design forces calculated in Step II. See Equation (4.3) below:

$$\phi R_n > \sum \gamma_i\ Q_i \qquad (4.3)$$

- Where, R_n is the calculated failure strength of the member
- ϕ is the resistance factor used to account for the reliability of the material behavior and equations for R_n
- Q_i is the nominal load
- γ_i is the load factor used to account for the variability in loading and to estimate the ultimate loading condition.

Example 1.1

Consider the building structure with the structural floor plan and elevation shown below. Estimate the wind loads acting on the structure when the wind blows in the east-west direction. The structure is located in Lansing.

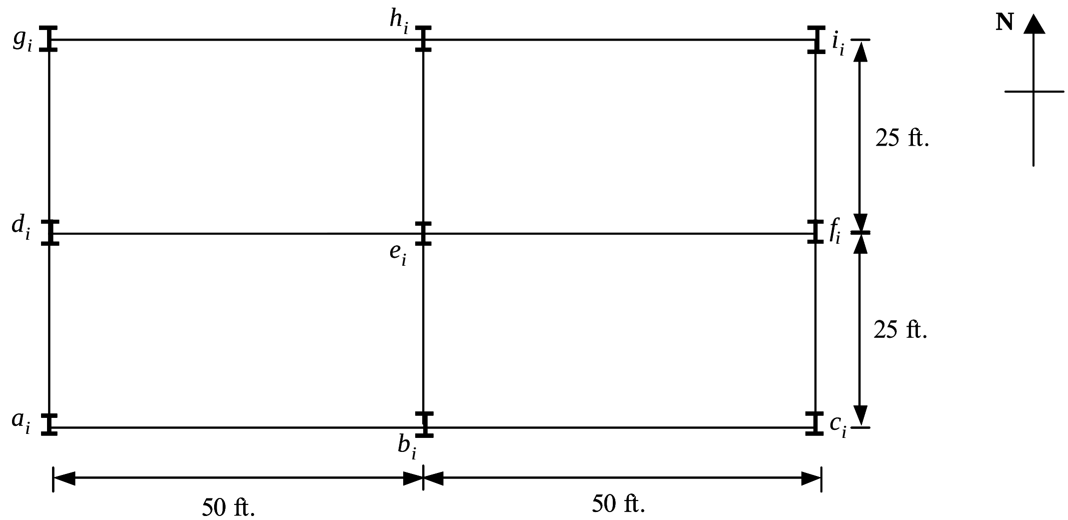


Figure 8. Structural floor plan

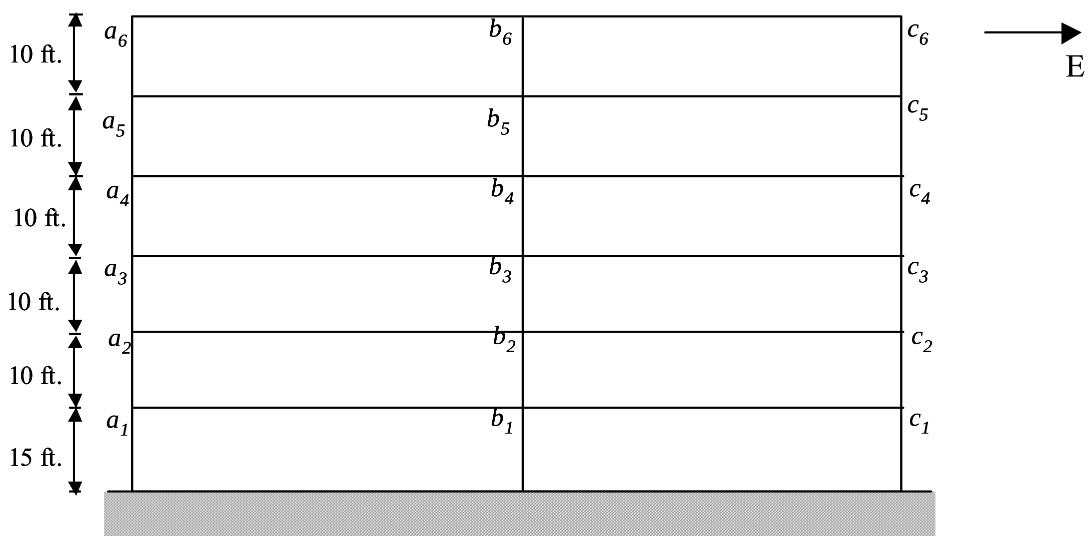


Figure 9. Structural elevation in east-west direction

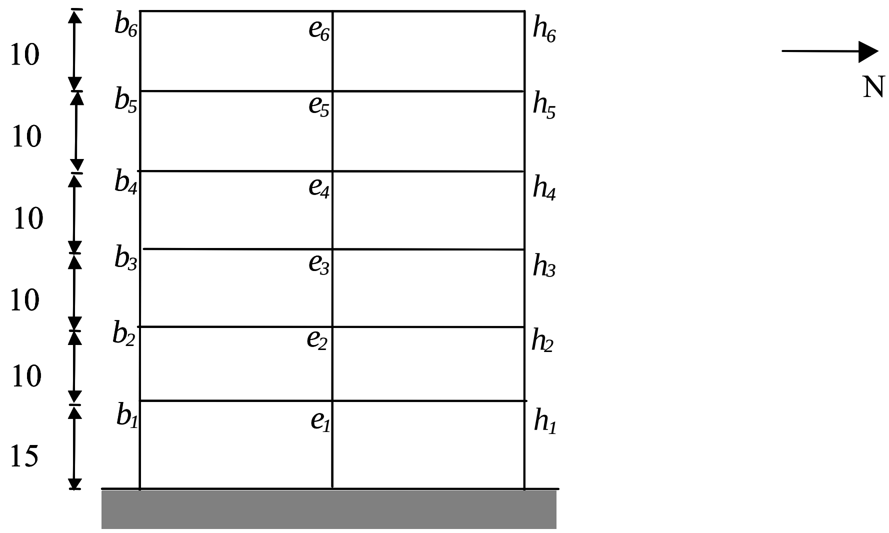


Figure 10. Structural elevation in north-south direction

- Velocity pressure (q_z)
  - K_d = directionality factor = 0.85
  - K_{zt} = topographic factor = 1.0
  - I = importance factor = 1.0
  - *K_h values for Exposure B, Case 2*

K_h	z
0.57	0 - 15
0.62	15 - 20
0.66	20 – 25
0.70	25 – 30
0.76	30 – 40
0.81	40 – 50
0.85	50 – 60
0.89	60 - 70

- $q_z = 0.00256\ K_z\ K_{zt}\ K_d\ V^2 I$
  - In Lansing V = 90 mph

- $q_z = 17.625\ K_z$ psf

- Wind pressure (p)

  - Gust factor = $G = 0.85$
  - For wind in east west direction; L/B = Length / width = 2.0
  - External pressure coefficient = $C_p = +0.8$ for windward walls
    $C_p = -0.3$ for leeward walls
    $C_p = -0.7$ for side walls
  - External pressure = $q\ G\ C_p$
  - External pressure on windward wall = $q_z\ GC_p = 17.625\ K_z \times 0.85 \times 0.8$
    $= 11.99\ K_z$ psf toward surface
  - External pressure on leeward wall = $q_h\ GC_p\ = 17.625\ K_{65} \times 0.85 \times (-0.3)$
    = 4.00 psf away from surface
  - External pressure on side wall = $q_h\ GC_p\ =17.625\ K_{65} \times 0.85 \times (-0.7)$
    = 9.33 psf away from surface
  - The external pressures on the structure are shown in Figures 11 and 12 below.

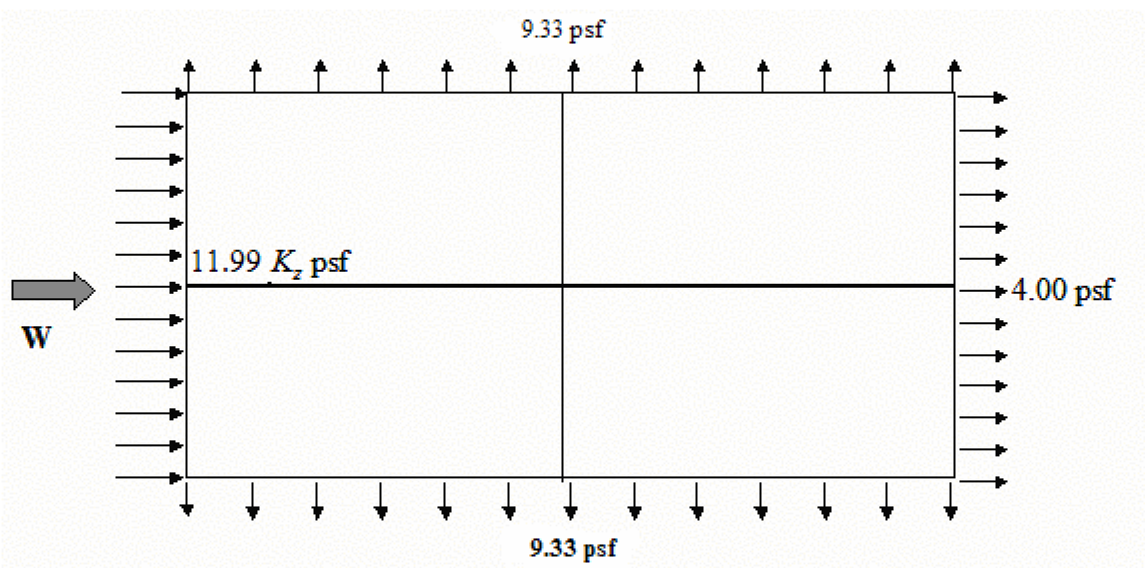


Figure 11. External pressures on structural plan

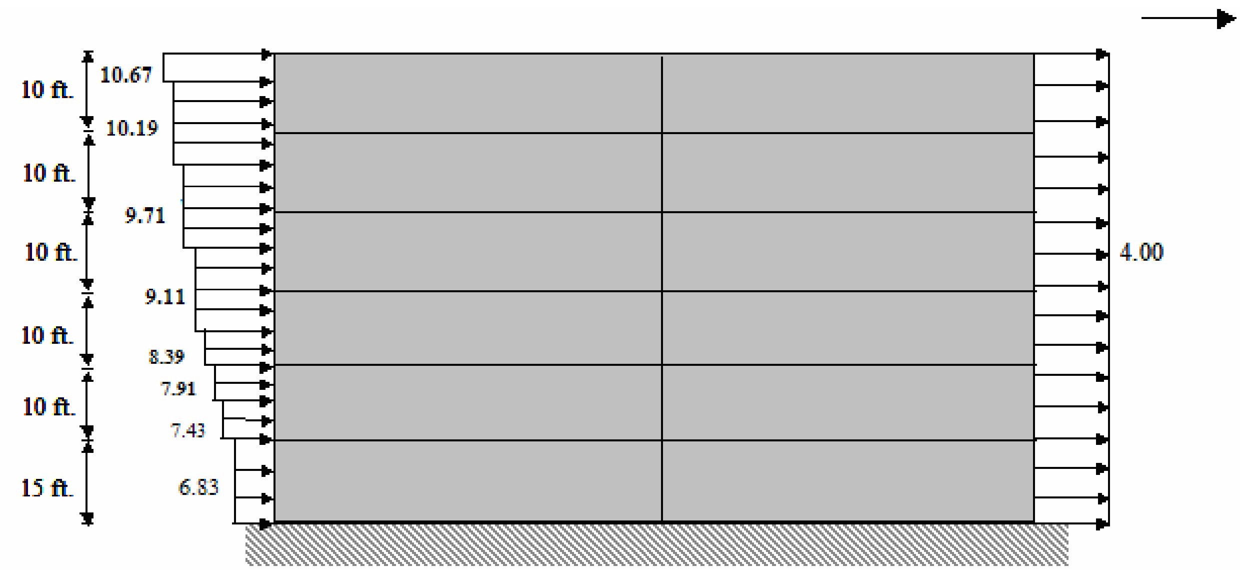


Figure 12. External pressure on structural elevation (east west)

- Internal pressure
  - $p = q\ GC_p - q_i\ GC_{pi}$
  - $q_i = q_h = 17.625\ K_{65} = 17.625 \times 0.89 = 15.69\ psf$
  - Enclosed building; $GC_{pi} = +0.18$ (acting toward surface)
    $GC_{pi} = -0.18$ (acting away from surface)
  - $q_i\ GC_{pi} = 2.82$ psf acting toward or away from surface
  - See Figure 13 (a) and (b) below

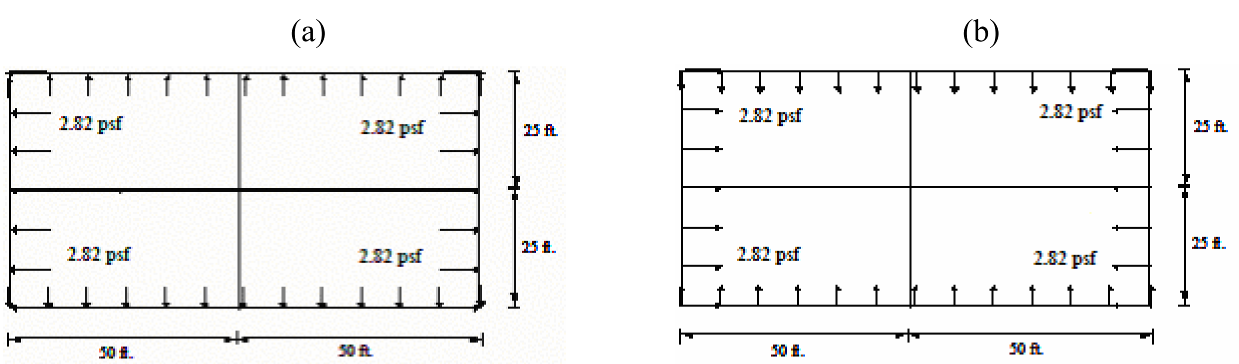


Figure 13. Internal pressure seen in structural plan

- Take the external pressure from Figure 11 and 12 and add to internal pressure from Figures 13 (a) and (b) to obtain the final pressure diagrams. Adding the internal pressure will not change the lateral forces in the structure.

(a) (b)

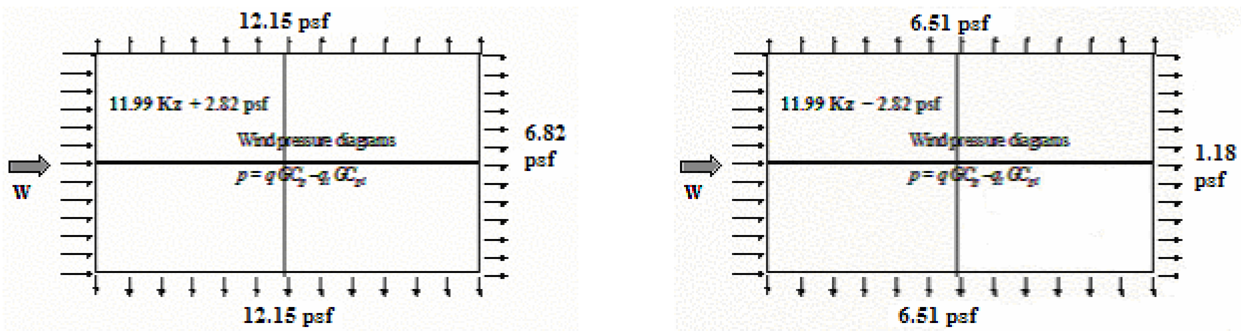


Figure 14. Resultant wind pressure diagrams including external and internal pressures

- Note: According to ASCE 7-98, the minimum wind design loading is equal to 10 lb/ft^2 multiplied by the area of the building projected on a vertical plane normal to assumed wind direction.

- The determined design wind loading is greater than the *minimum* value. Therefore, continue with estimated design wind loading.

Example 1.2 Determine the magnitude and distribution of live loading on the north-south frame b_i - e_i - h_i

- Step I: Determine relevant tributary and influence areas. Estimate live load reduction factors.

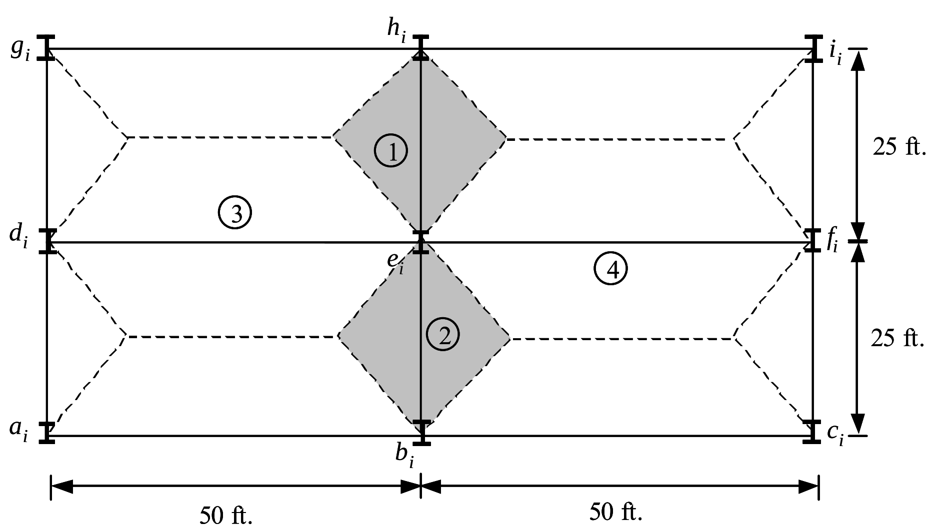
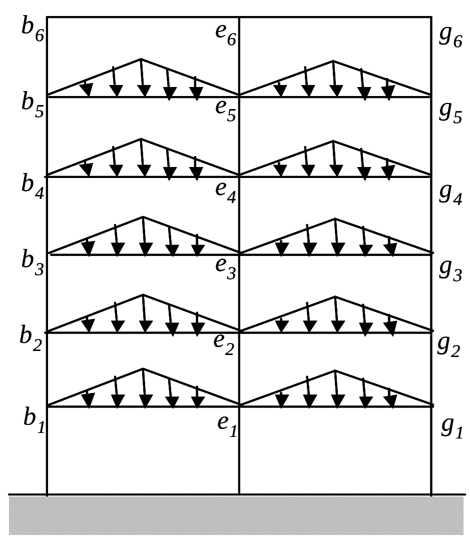


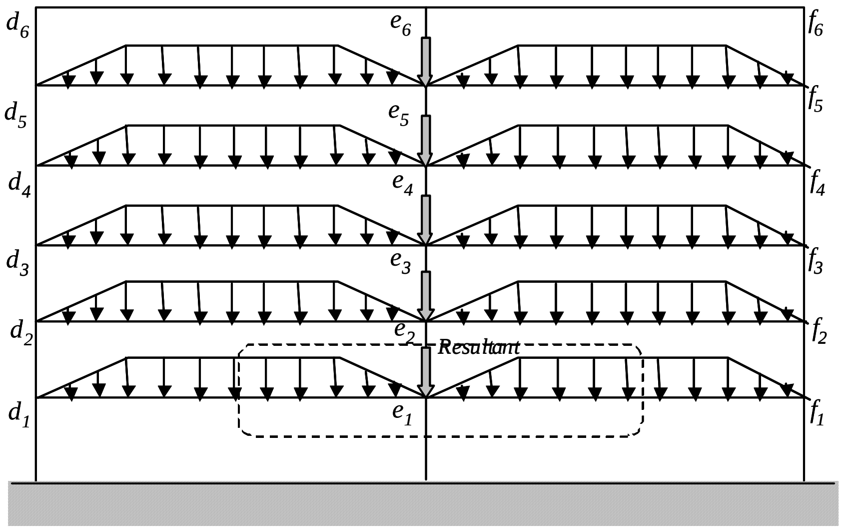
Table 1.1 Member tributary areas and minimum design live loading.

Member	Tributary area	K_{LL}	$L_o/L=0.25 + 4.57/(K_{LL}A_T)^{0.5}$	L_o/L min.
b_i-e_i	A_{T2} = ½ x 25.0 x 12.5 x 2 = 312.5 ft^2	2.0	0.4328	0.5
e_i - h_i	A_{T1} = ½ x 25.0 x 12.5 x 2 = 312.5 ft^2	2.0	0.4328	0.5
d_i - e_i	A_{T3} = ½ x 12.5 x 25.0 x 2 + 25.0 x 25.0 = 937.5 ft^2	2.0	0.36	0.5
e_i - f_i	A_{T4} = ½ x 12.5 x 25.0 x 2 + 25.0 x 25.0 = 937.5 ft^2	2.0	0.36	0.5
b_i	12.5 x 50.0 = 625.0 ft^2	4.0	0.34	0.4
e_i	25.0 x 50.0 = 1250.0 ft^2	4.0	0.3146	0.4
h_i	12.5 x 50.0 = 625 ft^2	4.0	0.34	0.4

- Step II. Estimate uniformly distributed loads



- Step III: Estimate live loading on columns from other frames than the one being investigated.



- Note: The minimum reduced live load for the column e_i from Table 1 = 0.40 L_o. However, the live loading on column e_i is being estimated using the reduced live loading on the beams. For consistency, make sure that the reduced beam live loading is not less than the reduced column live loading.

- Note: The wind pressures act on the sides of the building. The lateral forces acting on the frame are calculated using these wind pressures and the tributary area concept.

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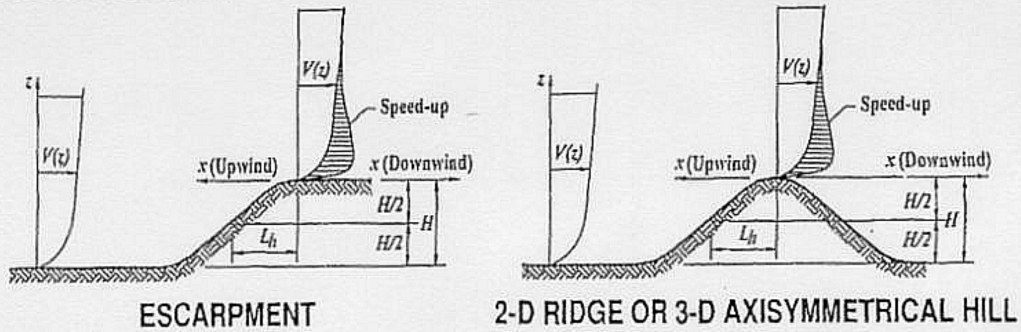
Topographic Factor, K_{zt}

Figure 6-2

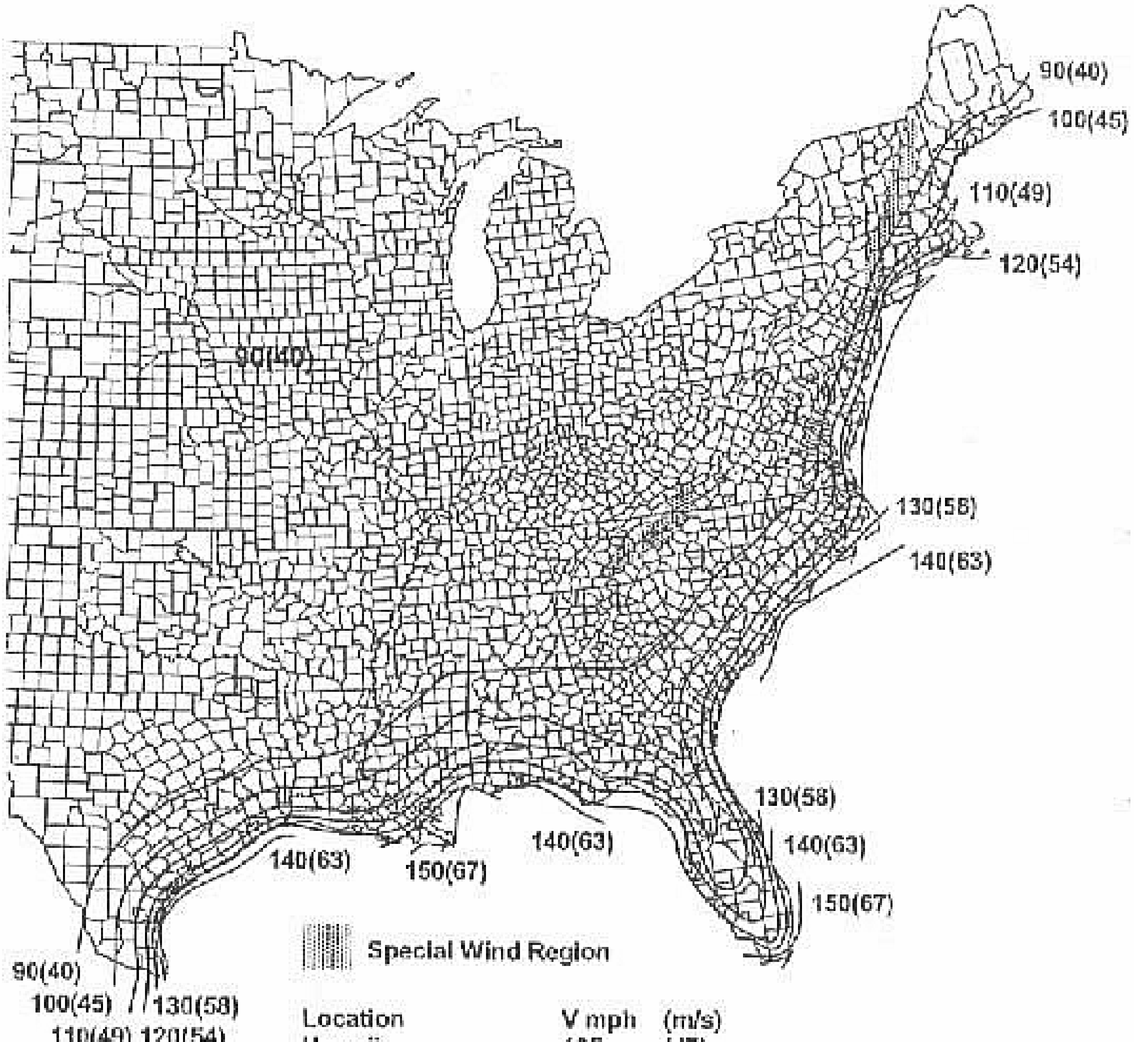


Topographic Multipliers for Exposure C										
	K_1 Multiplier				K_2 Multiplier			K_3 Multiplier		
H/L_h	2-D Ridge	2-D Escarp.	3-D Axisym. Hill	x/L_h	2-D Escarp.	All Other Cases	z/L_h	2-D Ridge	2-D Escarp.	3-D Axisym. Hill
0.20	0.29	0.17	0.21	0.00	1.00	1.00	0.00	1.00	1.00	1.00
0.25	0.36	0.21	0.26	0.50	0.88	0.67	0.10	0.74	0.78	0.67
0.30	0.43	0.26	0.32	1.00	0.75	0.33	0.20	0.55	0.61	0.45
0.35	0.51	0.30	0.37	1.50	0.63	0.00	0.30	0.41	0.47	0.30
0.40	0.58	0.34	0.42	2.00	0.50	0.00	0.40	0.30	0.37	0.20
0.45	0.65	0.38	0.47	2.50	0.38	0.00	0.50	0.22	0.29	0.14
0.50	0.72	0.43	0.53	3.00	0.25	0.00	0.60	0.17	0.22	0.09
				3.50	0.13	0.00	0.70	0.12	0.17	0.06
				4.00	0.00	0.00	0.80	0.09	0.14	0.04
							0.90	0.07	0.11	0.03
							1.00	0.05	0.08	0.02
							1.50	0.01	0.02	0.00
							2.00	0.00	0.00	0.00

Notes:

1. For values of H/L_h, x/L_h and z/L_h other than those shown, linear interpolation is permitted.
2. For $H/L_h > 0.5$, assume $H/L_h = 0.5$ for evaluating K_1 and substitute 2H for L_h for evaluating K_2 and K_3.
3. Multipliers are based on the assumption that wind approaches the hill or escarpment along the direction of maximum slope.
4. Notation:
   - H: Height of hill or escarpment relative to the upwind terrain, in feet (meters).
   - L_h: Distance upwind of crest to where the difference in ground elevation is half the height of hill or escarpment, in feet (meters).
   - K_1: Factor to account for shape of topographic feature and maximum speed-up effect.
   - K_2: Factor to account for reduction in speed-up with distance upwind or downwind of crest.
   - K_3: Factor to account for reduction in speed-up with height above local terrain.
   - x: Distance (upwind or downwind) from the crest to the building site, in feet (meters).
   - z: Height above local ground level, in feet (meters).
   - μ: Horizontal attenuation factor.
   - γ: Height attenuation factor.

Figure showing the Wind Speed of Eastern US. (ASCE 7 – 98 pg. 35)



Location	V mph	(m/s)
Hawaii	105	(47)
Puerto Rico	145	(65)
Guam	170	(76)
Virgin Islands	145	(65)
American Samoa	125	(56)

Notes:
1. Values are nominal design 3-second gust wind speeds in miles per hour (m/s) at 33 ft (10 m) above ground for Exposure C category.
2. Linear interpolation between wind contours is permitted.
3. Islands and coastal areas outside the last contour shall use the last wind speed contour of the coastal area.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.

FIGURE 6-1. *(Continued)*

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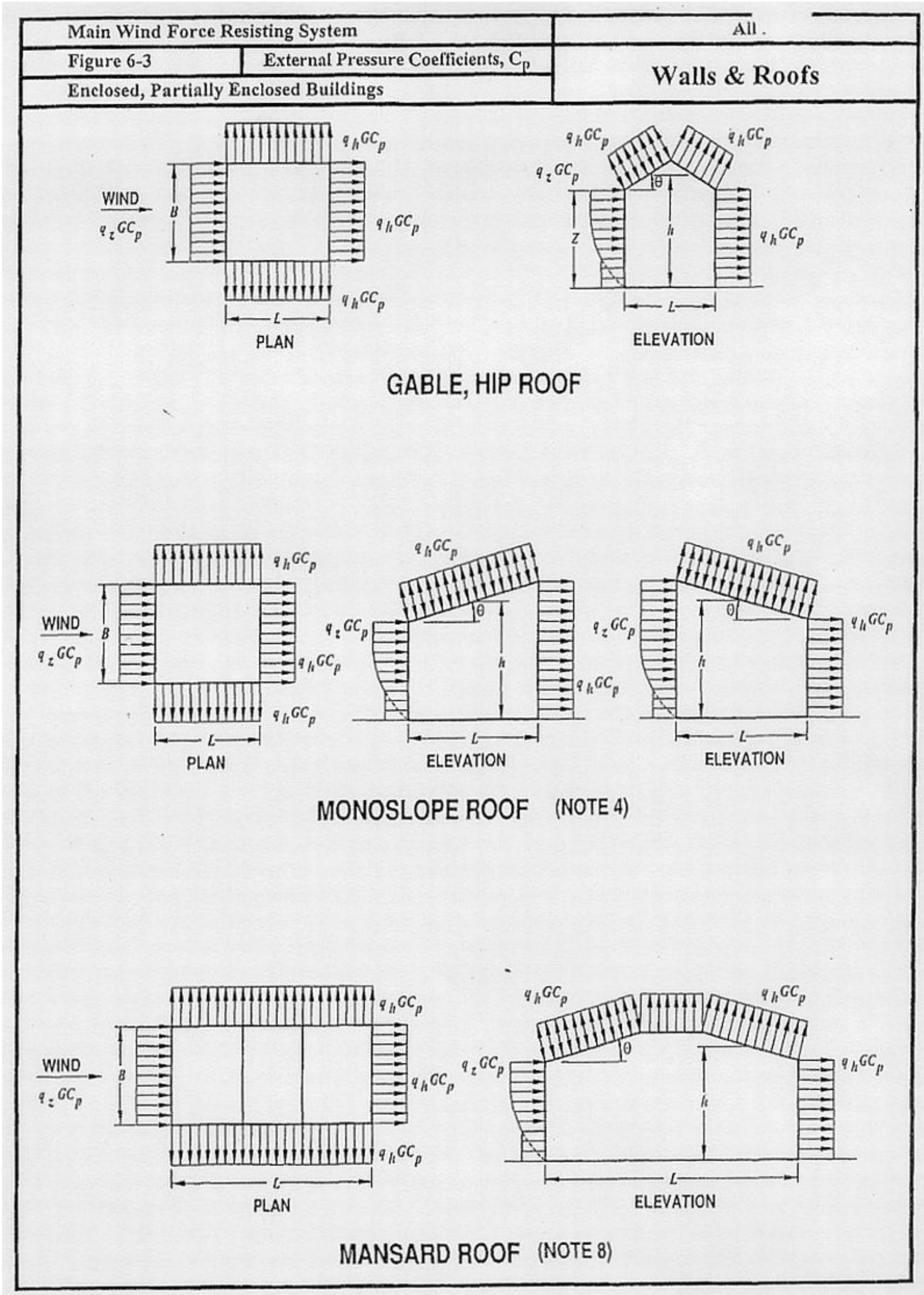

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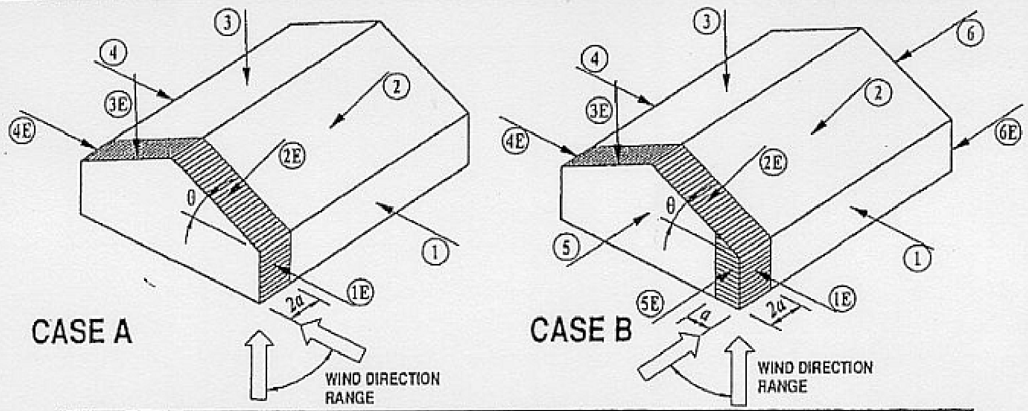
Main Wind Force Resisting System		h ≤ 60 ft.
Figure 6-4	External Pressure Coefficients, GC_{pf}	**Walls & Gable Roof**
Enclosed, Partially Enclosed Buildings		



Roof Angle θ (degrees)	CASE A							
	Building Surface							
	1	2	3	4	1E	2E	3E	4E
0-5	0.40	-0.69	-0.37	-0.29	0.61	-1.07	-0.53	-0.43
20	0.53	-0.69	-0.48	-0.43	0.80	-1.07	-0.69	-0.64
30-45	0.56	0.21	-0.43	-0.37	0.69	0.27	-0.53	-0.48
90	0.56	0.56	-0.37	-0.37	0.69	0.69	-0.48	-0.48

Roof Angle θ (degrees)	CASE B											
	Building Surface											
	1	2	3	4	5	6	1E	2E	3E	4E	5E	6E
0-90	-0.45	-0.69	-0.37	-0.45	0.40	-0.29	-0.48	-1.07	-0.53	-0.48	0.61	-0.43

Notes:

1. Case A and Case B are required as two separate loading conditions to generate the wind actions, including torsion, to be resisted by the main wind-force resisting system.
2. To obtain the critical wind actions, the building shall be rotated in 90° degree increments so that each corner in turn becomes the windward corner while the loading patterns in the sketches remain fixed. For the design of structural systems providing lateral resistance in the direction parallel to the ridge line, Case A shall be based on $\theta = 0°$.
3. Plus and minus signs signify pressures acting toward and away from the surfaces, respectively.
4. For Case A loading the following restrictions apply:
   a. The roof pressure coefficient GC_{pf}, when negative in Zone 2, shall be applied in Zone 2 for a distance from the edge of roof equal to 0.5 times the horizontal dimensions of the building measured perpendicular to the eave line or $2.5h$, whichever is less; the remainder of Zone 2 extending to the ridge line shall use the pressure coefficient GC_{pf} for Zone 3.
   b. Except for moment-resisting frames, the total horizontal shear shall not be less than that determined by neglecting wind forces on roof surfaces.
5. Combinations of external and internal pressures (see Table 6-7) shall be evaluated as required to obtain the most severe loadings.
6. For values of θ other than those shown, linear interpolation is permitted.
7. Notation:
   a: 10 percent of least horizontal dimension or 0.4h, whichever is smaller, but not less than either 4% of least horizontal dimension or 3 ft (1 m).
   h: Mean roof height, in feet (meters), except that eave height shall be used for $\theta \leq 10°$.
   θ: Angle of plane of roof from horizontal, in degrees.

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Main Wind Force Resisting System		All h
Figure 6-3 (con't)	External Pressure Coefficients, C_p	Walls & Roofs
Enclosed, Partially Enclosed Buildings		

Wall Pressure Coefficients, C_p

Surface	L/B	C_p	Use With
Windward Wall	All values	0.8	q_z
Leeward Wall	0-1	-0.5	q_h
	2	-0.3	
	≥4	-0.2	
Side Wall	All values	-0.7	q_h

Roof Pressure Coefficients, C_p, for use with q_h

Wind Direction	Windward									Leeward		
	Angle, θ (degrees)									Angle, θ (degrees)		
	h/L	10	15	20	25	30	35	45	≥60#	10	15	≥20
Normal to ridge for θ ≥ 10°	≤0.25	-0.7	-0.5 0.0*	-0.3 0.2	-0.2 0.3	-0.2 0.3	0.0* 0.4	0.4	0.01 θ	-0.3	-0.5	-0.6
	0.5	-0.9	-0.7	0.4 0.0*	-0.3 0.2	-0.2 0.2	-0.2 0.3	0.0* 0.4	0.01 θ	-0.5	-0.5	-0.6
	≥1.0	-1.3**	-1.0	-0.7	-0.5 0.0*	-0.3 0.2	-0.2 0.2	0.0* 0.3	0.01 θ	-0.7	-0.6	-0.6

Wind Direction	h/L	Horiz distance from windward edge	C_p
Normal to ridge for θ < 10 and Parallel to ridge for all θ	≤ 0.5	0 to h/2	-0.9
		h/2 to h	-0.9
		h to 2 h	-0.5
		> 2h	-0.3
	≥ 1.0	0 to h/2	-1.3**
		> h/2	-0.7

*Value is provided for interpolation purposes.

**Value can be reduced linearly with area over which it is applicable as follows

Area (sq ft)	Reduction Factor
≤ 100 (9.29 sq m)	1.0
200 (23.23 sq m)	0.9
≥ 1000 (92.9 sq m)	0.8

Notes:

1. Plus and minus signs signify pressures acting toward and away from the surfaces, respectively
2. Linear interpolation is permitted for values of *L/B*, *h/L* and θ other than shown. Interpolation shall only be carried out between values of the same sign. Where no value of the same sign is given, assume 0.0 for interpolation purposes.
3. Where two values of C_p are listed, this indicates that the windward roof slope is subjected to either positive or negative pressures and the roof structure shall be designed for both conditions. Interpolation for intermediate ratios of h/L in this case shall only be carried out between C_p values of like sign.
4. For monoslope roofs, entire roof surface is either a windward or leeward surface.
5. For flexible buildings use appropriate G_f as determined by rational analysis.
6. Refer to Table 6-8 for arched roofs.
7. Notation:
   B: Horizontal dimension of building, in feet (meter), measured normal to wind direction.
   L: Horizontal dimension of building, in feet (meter), measured parallel to wind direction.
   h: Mean roof height in feet (meters), except that eave height shall be used for θ ≤ 10 degrees.
   z: Height above ground, in feet (meters).
   G: Gust effect factor.
   q_z, q_h: Velocity pressure, in pounds per square foot (N/m²), evaluated at respective height.
   θ: Angle of plane of roof from horizontal, in degrees.
8. For mansard roofs, the top horizontal surface and leeward inclined surface shall be treated as leeward surfaces from the table.

#For roof slopes greater than 80°, use $C_p = 0.8$

Importance factor ASCE 7 – 98 pg. 55

Category	Non-Hurricane Prone Regions and Hurricane Prone Regions with V = 85-100 mph and Alaska	Hurricane Prone Regions with V > 100 mph
I	0.87	0.77
II	1.00	1.00
III	1.15	1.15
IV	1.15	1.15

Note:

1. The building and structure classification categories are listed in Table 1-1.

wind directionality factor asce 7 – 98 pg.

Structure Type	Directionality Factor K_d*
Buildings	
Main Wind Force Resisting System	0.85
Components and Cladding	0.85
Arched Roofs	0.85
Chimneys, Tanks, and Similar Structures	
Square	0.90
Hexagonal	0.95
Round	0.95
Solid Signs	0.85
Open Signs and Lattice Framework	0.85
Trussed Towers	
Triangular, square, rectangular	0.85
All other cross sections	0.95

*Directionality Factor K_d has been calibrated with combinations of loads specified in Section 2. This factor shall only be applied when used in conjunction with load combinations specified in 2.3 and 2.4.

velocity pressure exposure coefficient

Velocity Pressure Exposure Coefficients, K_h and K_z

Table 6-5

Height above ground level, z		Exposure (Note 1)					
		A		B		C	D
ft	(m)	Case 1	Case 2	Case 1	Case 2	Cases 1 & 2	Cases 1 & 2
0-15	(0-4.6)	0.68	0.32	0.70	0.57	0.85	1.03
20	(6.1)	0.68	0.36	0.70	0.62	0.90	1.08
25	(7.6)	0.68	0.39	0.70	0.66	0.94	1.12
30	(9.1)	0.68	0.42	0.70	0.70	0.98	1.16
40	(12.2)	0.68	0.47	0.76	0.76	1.04	1.22
50	(15.2)	0.68	0.52	0.81	0.81	1.09	1.27
60	(18)	0.68	0.55	0.85	0.85	1.13	1.31
70	(21.3)	0.68	0.59	0.89	0.89	1.17	1.34
80	(24.4)	0.68	0.62	0.93	0.93	1.21	1.38
90	(27.4)	0.68	0.65	0.96	0.96	1.24	1.40
100	(30.5)	0.68	0.68	0.99	0.99	1.26	1.43
120	(36.6)	0.73	0.73	1.04	1.04	1.31	1.48
140	(42.7)	0.78	0.78	1.09	1.09	1.36	1.52
160	(48.8)	0.82	0.82	1.13	1.13	1.39	1.55
180	(54.9)	0.86	0.86	1.17	1.17	1.43	1.58
200	(61.0)	0.90	0.90	1.20	1.20	1.46	1.61
250	(76.2)	0.98	0.98	1.28	1.28	1.53	1.68
300	(91.4)	1.05	1.05	1.35	1.35	1.59	1.73
350	(106.7)	1.12	1.12	1.41	1.41	1.64	1.78
400	(121.9)	1.18	1.18	1.47	1.47	1.69	1.82
450	(137.2)	1.24	1.24	1.52	1.52	1.73	1.86
500	(152.4)	1.29	1.29	1.56	1.56	1.77	1.89

Notes:

1. **Case 1:** a. All components and cladding.
   b. Main wind force resisting system in low-rise buildings designed using Figure 6-4.

   Case 2: a. All main wind force resisting systems in buildings except those in low-rise buildings designed using Figure 6-4.
   b. All main wind force resisting systems in other structures.
2. The velocity pressure exposure coefficient K_z may be determined from the following formula:

   For 15 ft. $\leq z \leq z_g$: $K_z = 2.01\ (z/z_g)^{2/\alpha}$

   For $z <$ 15 ft.: $K_z = 2.01\ (15/z_g)^{2/\alpha}$

   Note: z shall not be taken less than 100 feet for Case 1 in exposure A or less than 30 feet for Case 1 in exposure B.
3. α and z_g are tabulated in Table 6-4.
4. Linear interpolation for intermediate values of height z is acceptable.
5. Exposure categories are defined in 6.5.6.

internal pressure coefficient for buildings

Enclosure Classification	GC_{pi}
Open Buildings	0.00
Partially Enclosed Buildings	+0.55 -0.55
Enclosed Buildings	+0.18 -0.18

Notes:

1. Plus and minus signs signify pressures acting toward and away from the internal surfaces.
2. Values of GC_{pi} shall be used with q_z or q_h as specified in 6.5.12.
3. Two cases shall be considered to determine the critical load requirements for the appropriate condition:

   (i) a positive value of GC_{pi} applied to all internal surfaces
   (ii) a negative value of GC_{pi} applied to all internal surfaces

Chapter 2. Design of Beams – Flexure and Shear

2.1 Section force-deformation response & Plastic Moment (M_p)

- A beam is a structural member that is subjected primarily to transverse loads and negligible axial loads.
- The transverse loads cause internal shear forces and bending moments in the beams as shown in Figure 1 below.

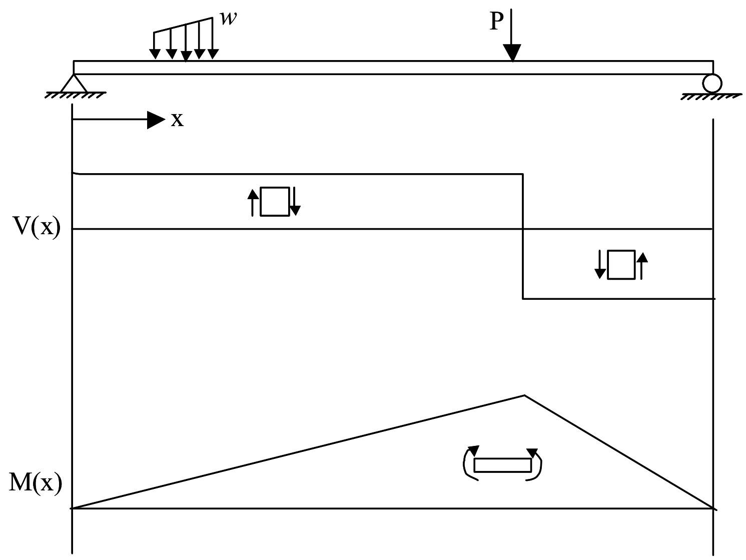
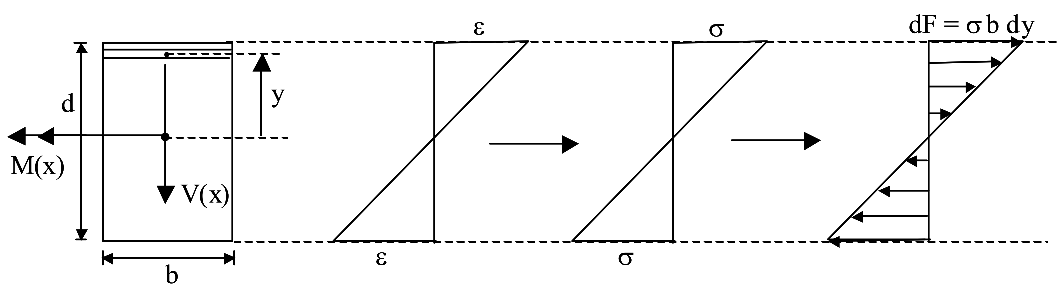


Figure 1. Internal shear force and bending moment diagrams for transversely loaded beams.

- These internal shear forces and bending moments cause longitudinal axial stresses and shear stresses in the cross-section as shown in the Figure 2 below.



Curvature = $\phi = 2\varepsilon/d$ $\quad$ $F = \int_{-d/2}^{+d/2} \sigma \, b \, dy$ $\quad$ $M = \int_{-d/2}^{+d/2} \sigma \, b \, dy \; y$ (Planes remain plane)

Figure 2. Longitudinal axial stresses caused by internal bending moment.

- Steel material follows a typical stress-strain behavior as shown in Figure 3 below.

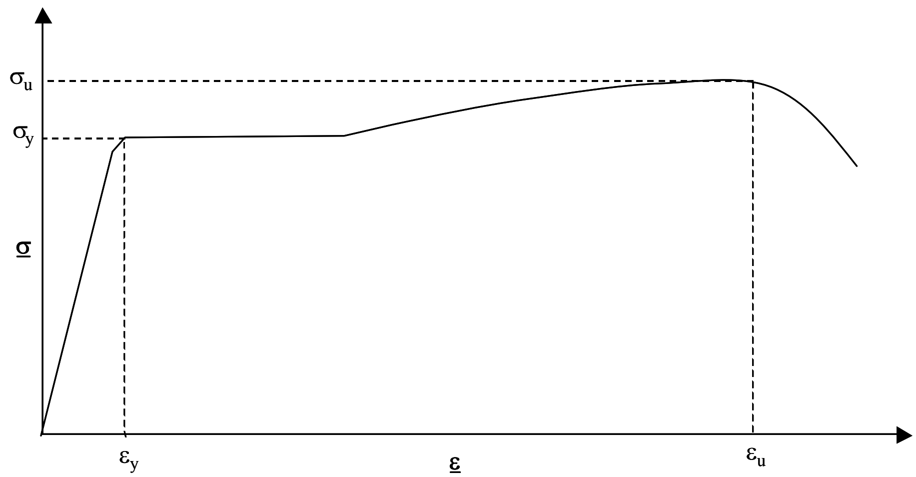
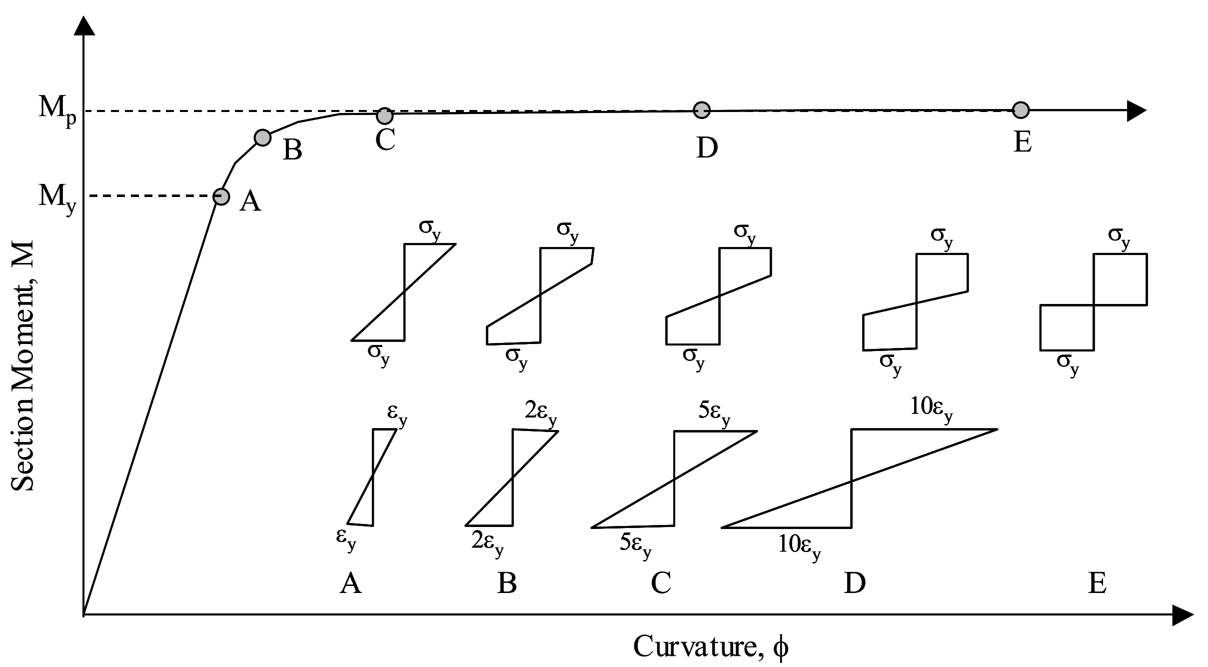


Figure 3. Typical steel stress-strain behavior.

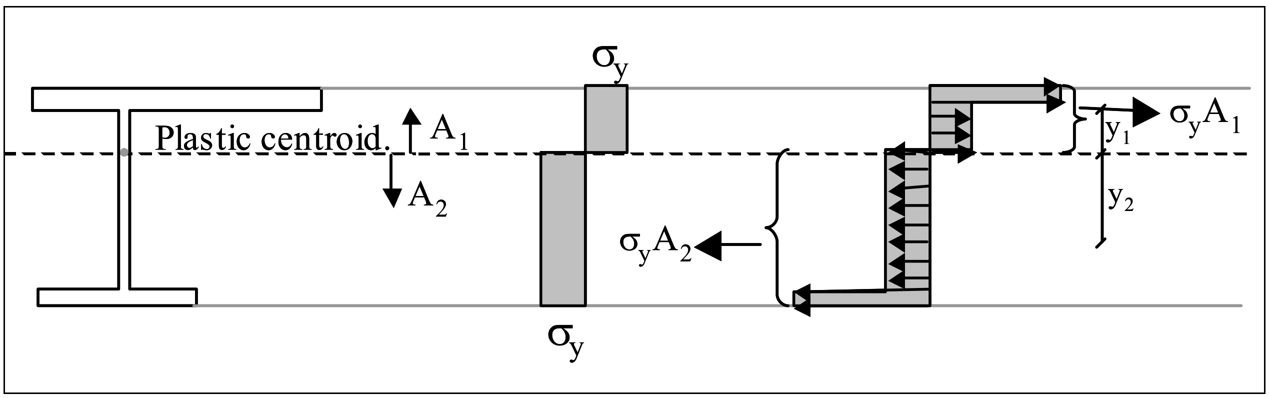
- If the steel stress-strain curve is approximated as a bilinear elasto-plastic curve with yield stress equal to σ_y, then the section Moment - Curvature (M-ϕ) response for monotonically increasing moment is given by Figure 4.



A: Extreme fiber reaches ε_y B: Extreme fiber reaches $2\varepsilon_y$ C: Extreme fiber reaches $5\varepsilon_y$
D: Extreme fiber reaches $10\varepsilon_y$ E: Extreme fiber reaches infinite strain

Figure 4. Section Moment - Curvature (M-ϕ) behavior.

- In Figure 4, M_y is the moment corresponding to first yield and M_p is the plastic moment capacity of the cross-section.
  - The ratio of M_p to M_y is called as the shape factor *f* for the section.
  - For a rectangular section, *f* is equal to 1.5. For a wide-flange section, *f* is equal to 1.1.
- Calculation of M_p: Cross-section subjected to either $+\sigma_y$ or $-\sigma_y$ at the plastic limit. See Figure 5 below.



(a) General cross-section (b) Stress distribution (c) Force distribution

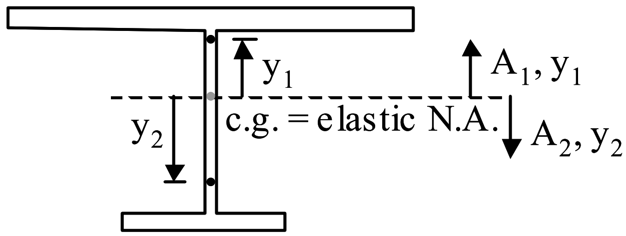
$$F = \sigma_y A_1 - \sigma_y A_2 = 0$$
$$\therefore A_1 = A_2 = A/2$$
$$\therefore M = \sigma_y \frac{A}{2} \times (y_1 + y_2)$$
$$\text{Where, } y_1 = \text{centroid of } A_1$$
$$y_2 = \text{centroid of } A_2$$

(d) Equations

Figure 5. Plastic centroid and M_p for general cross-section.

- The plastic centroid for a general cross-section corresponds to the axis about which the total area is equally divided, i.e., $A_1 = A_2 = A/2$
  - The plastic centroid is not the same as the elastic centroid or center of gravity (c.g.) of the cross-section.

- As shown below, the c.g. is defined as the axis about which $A_1y_1 = A_2y_2$.



About the c.g. $A_1y_1 = A_2y_2$

- For a cross-section with at-least one axis of symmetry, the neutral axis corresponds to the centroidal axis in the elastic range. However, at M_p, the neutral axis will correspond to the plastic centroidal axis.

- **For a doubly symmetric cross-section, the elastic and the plastic centroid lie at the same point.**
- $M_p = \sigma_y \times A/2 \times (y_1+y_2)$
- As shown in Figure 5, y_1 and y_2 are the distance from the plastic centroid to the centroid of area A_1 and A_2, respectively.
- $A/2 \times (y_1+y_2)$ is called **Z,** the plastic section modulus of the cross-section. Values for Z are tabulated for various cross-sections in the properties section of the LRFD manual.
- $\phi M_p = 0.90\ Z\ F_y$ - See Spec. F1.1

where,

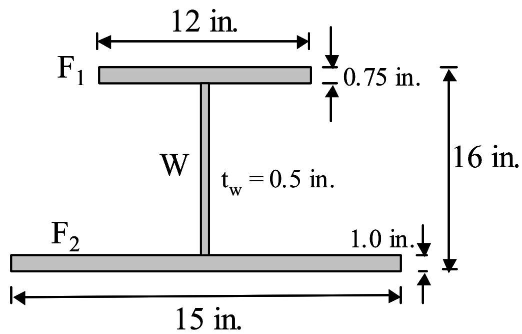
M_p = plastic moment, which must be ≤ 1.5 M_y for homogenous cross-sections

M_y = moment corresponding to onset of yielding at the extreme fiber from an elastic stress distribution = F_y S for homogenous cross-sections and = F_{yf} S for hybrid sections.

Z = plastic section modulus from the Properties section of the AISC manual.

S = elastic section modulus, also from the Properties section of the AISC manual.

Example 2.1 Determine the elastic section modulus, S, plastic section modulus, Z, yield moment, M_y, and the plastic moment M_p, of the cross-section shown below. What is the design moment for the beam cross-section. Assume 50 ksi steel.



- A_g = 12 x 0.75 + (16 - 0.75 - 1.0) x 0.5 + 15 x 1.0 = 31.125 in^2

  A_{f1} = 12 x 0.75 = 9 in^2

  A_{f2} = 15 x 1.0 = 15.0 in^2

  A_w = 0.5 x (16 - 0.75 - 1.0) = 7.125 in^2

- distance of elastic centroid from bottom = $\bar{y}$

  $$\bar{y} = \frac{9\times(16-0.75/2)+7.125\times8.125+15\times0.5}{31.125} = 6.619 \text{ in.}$$

  $I_x = 12\times0.75^3/12 + 9.0\times9.006^2 + 0.5\times14.25^3/12 + 7.125\times1.506^2 + 15.0\times1^3/12 +$

  $15\times6.119^2 = 1430 \text{ in}^4$

  $S_x = I_x / (16\text{-}6.619) = 152.43 \text{ in}^3$

  $M_{y\text{-}x} = F_y\, S_x$ = 7621.8 kip-in. = 635.15 kip-ft.

- distance of plastic centroid from bottom = $\bar{y}_p$

  $$\therefore\quad 15.0\times1.0+0.5\times(\bar{y}_p-1.0) = \frac{31.125}{2} = 15.5625$$

  $$\therefore\quad \bar{y}_p = 2.125 \text{ in.}$$

y_1=centroid of top half-area about plastic centroid = $\dfrac{9\times 13.5 + 6.5625\times 6.5625}{15.5625} = 10.5746$ in.

y_2=centroid of bottom half-area about plas. cent. = $\dfrac{0.5625\times 0.5625 + 15.0\times 1.625}{15.5625} = 1.5866$ in.

Z_x = A/2 x (y_1 + y_2) = 15.5625 x (10.5746 + 1.5866) = 189.26 in^3

$M_{p\text{-}x}$ = Z_x F_y = 189.26 x 50 = 9462.93 kip-in. = 788.58 kip-ft.

- Design strength according to AISC Spec. F1.1= $\phi_b M_p$= 0.9 x 788.58 = 709.72 kip-ft.
- Check = $M_p \leq 1.5\ M_y$

  Therefore, 788.58 kip-ft. < 1.5 x 635.15 = 949.725 kip-ft. - OK!

- Reading Assignment

2.2 Flexural Deflection of Beams – Serviceability

- Steel beams are designed for the factored design loads. The moment capacity, i.e., the factored moment strength ($\phi_b M_n$) should be greater than the moment (M_u) caused by the factored loads.
- A *serviceable* structure is one that performs satisfactorily, not causing discomfort or perceptions of unsafety for the occupants or users of the structure.
  - For a beam, being serviceable usually means that the deformations, primarily the vertical slag, or deflection, must be limited.
  - The maximum deflection of the designed beam is checked at the service-level loads. The deflection due to service-level loads must be less than the specified values.
- The AISC Specification gives little guidance other than a statement in Chapter L, "*Serviceability Design Considerations*," that deflections should be checked. Appropriate limits for deflection can be found from the governing building code for the region.
- The following values of deflection are typical maximum allowable total (service dead load plus service live load) deflections.
  - Plastered floor construction – L/360
  - Unplastered floor construction – L/240
  - Unplastered roof construction – L/180
- In the following examples, we will assume that local buckling and lateral-torsional buckling are not controlling limit states, i.e, the beam section is compact and laterally supported along the length.

Example 2.2 Design a simply supported beam subjected to uniformly distributed dead load of 450 lbs/ft. and a uniformly distributed live load of 550 lbs/ft. The dead load does not include the self-weight of the beam.

- **Step I.** Calculate the factored design loads (without self-weight).

  $w_U = 1.2\ w_D + 1.6\ w_L = 1.42$ kips / ft.

  $M_U = w_u\ L^2 / 8 = 1.42 \times 30^2 / 8 = 159.75$ kip-ft.

- **Step II.** Select the lightest section from the AISC Manual design tables.

  From page ____________ of the AISC manual, select **W16 x 26** made from 50 ksi steel with

  $\phi_b M_p = 166.0$ kip-ft.

- **Step III.** Add self-weight of designed section and check design

  $w_{sw} = 26$ lbs/ft

  Therefore, $w_D = 476$ lbs/ft = 0.476 lbs/ft.

  $w_u = 1.2 \times 0.476 + 1.6 \times 0.55 = 1.4512$ kips/ft.

  Therefore, $M_u = 1.4512 \times 30^2 / 8 = 163.26$ kip-ft. $< \phi_b M_p$ of W16 x 26.

  OK!

- **Step IV.** Check deflection at service loads.

  $w = 0.45 + 0.026 + 0.55$ kips/ft. = 1.026 kips/ft.

  $\Delta = 5\ w\ L^4 / (384\ E\ I_x) = 5 \times (1.026/12) \times (30 \times 12)^4 / (384 \times 29000 \times 301)$

  $\Delta = 2.142$ in. $> L/360$ - for plastered floor construction

- **Step V.** Redesign with service-load deflection as design criteria

  $L/360 = 1.0$ in. $> 5\ w\ L^4/(384\ E\ I_x)$

  Therefore, $I_x > 644.8\ in^4$

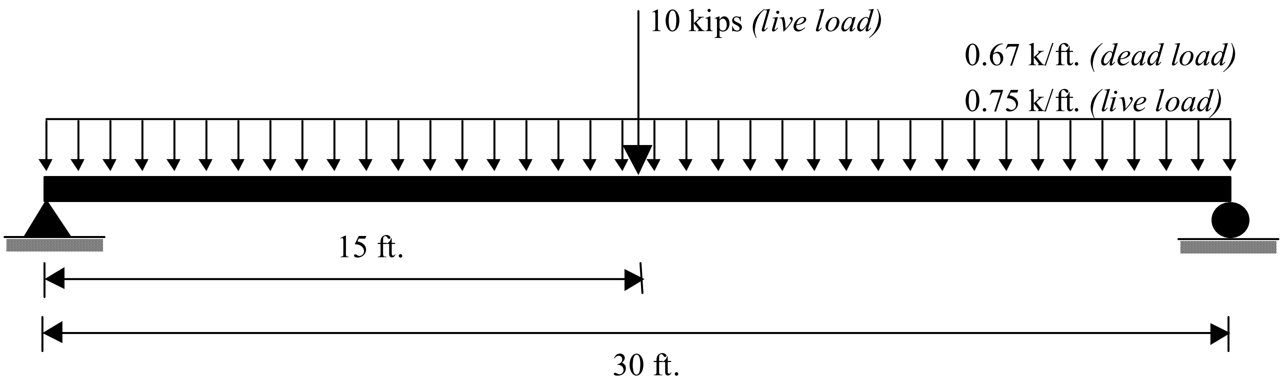
Select the section from the *moment of inertia* selection tables in the AISC manual. See page – select **W21 x 44.**

W21 x 44 with $\mathbf{I_x}$ = 843 in^4 and $\boldsymbol{\phi_b M_p}$ = 358 kip-ft. (50 ksi steel).

Deflection at service load = Δ = <u>0.765 in.</u> < L/360 - **<u>OK!</u>**

Note that the serviceability design criteria controlled the design and the section

Example 2.3 Design the beam shown below. The unfactored dead and live loads are shown in the Figure.



- **Step I.** Calculate the factored design loads (without self-weight).

  $w_u = 1.2\ w_D + 1.6\ w_L = 1.2 \times 0.67 + 1.6 \times 0.75 = 2.004$ kips / ft.

  $P_u = 1.2\ P_D + 1.6\ P_L = 1.2 \times 0 + 1.6 \times 10 = 16.0$ kips

  $M_u = w_U\ L^2 / 8 + P_U\ L / 4 = 225.45 + 120 = 345.45$ kip-ft.

- **Step II.** Select the lightest section from the AISC Manual design tables.

  From page ____________ of the AISC manual, select **W21 x 44** made from 50 ksi steel with $\phi_b M_p$ = 358.0 kip-ft.

  Self-weight = w_{sw} = 44 lb/ft.

- **Step III.** Add self-weight of designed section and check design

  $w_D = 0.67 + 0.044 = 0.714$ kips/ft

  $w_u = 1.2 \times 0.714 + 1.6 \times 0.75 = 2.0568$ kips/ft.

  Therefore, $M_u = 2.0568 \times 30^2 / 8 + 120 = 351.39$ kip-ft. $< \phi_b M_p$ of W21 x 44.

  <u>OK!</u>

- **Step IV.** Check deflection at service loads.

  Service loads

  – Distributed load = w = 0.714 + 0.75 = 1.464 kips/ft.

  – Concentrated load = P = D + L = 0 + 10 kips = 10 kips

  Deflection due to uniform distributed load = $\Delta_d = 5\, w\, L^4 / (384\, EI)$

  Deflection due to concentrated load = $\Delta_c = P\, L^3 / (48\, EI)$

Therefore, service-load deflection = $\Delta = \Delta_d + \Delta_c$

$\Delta = 5 \times 1.464 \times 360^4 / (384 \times 29000 \times 12 \times 843) + 10 \times 360^3 / (48 \times 29000 \times 843)$

$\Delta = 1.0914 + 0.3976 = 1.49$ in.

Assuming unplastered floor construction, $\Delta_{max} = L/240 = 360/240 = 1.5$ in.

Therefore, $\Delta < \Delta_{max}$ **<u>- OK!</u>**

2.3 Local buckling of beam section – Compact and Non-compact

- M_p, the plastic moment capacity for the steel shape, is calculated by assuming a plastic stress distribution (+ or - σ_y) over the cross-section.
- The development of a plastic stress distribution over the cross-section can be hindered by two different length effects:

  (1) *Local buckling* of the individual plates (flanges and webs) of the cross-section before they develop the compressive yield stress σ_y.

  (2) *Lateral-torsional buckling* of the unsupported length of the beam / member before the cross-section develops the plastic moment M_p.

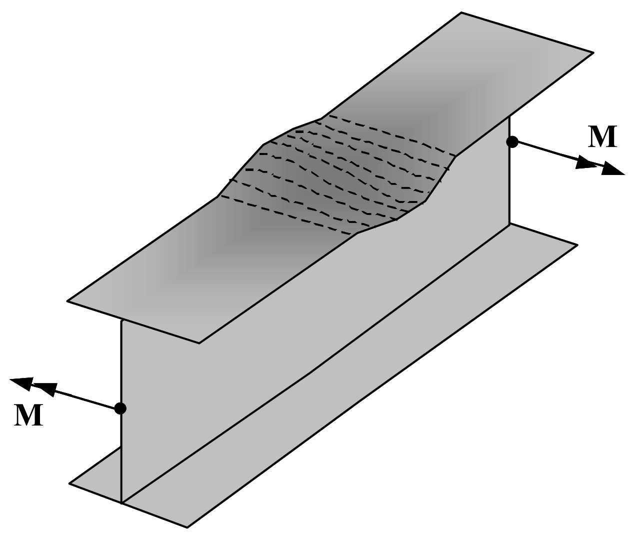


Figure 7. Local buckling of flange due to compressive stress (σ)

- The analytical equations for local buckling of steel plates with various edge conditions and the results from experimental investigations have been used to develop limiting slenderness ratios for the individual plate elements of the cross-sections.
- See Spec. B5 (page 16.1 – 12), Table B5.1 (16.1-13) and Page 16.1-183 of the AISC-manual
- Steel sections are classified as compact, non-compact, or slender depending upon the slenderness (λ) ratio of the individual plates of the cross-section.

- *Compact section* if all elements of cross-section have $\lambda \leq \lambda_p$
- *Non-compact sections* if any one element of the cross-section has $\lambda_p \leq \lambda \leq \lambda_r$
- *Slender section* if any element of the cross-section has $\lambda_r \leq \lambda$

- It is important to note that:
  - If $\lambda \leq \lambda_p$, then the individual plate element can develop and sustain σ_y for large values of ε before local buckling occurs.
  - If $\lambda_p \leq \lambda \leq \lambda_r$, then the individual plate element can develop σ_y but cannot sustain it before local buckling occurs.
  - If $\lambda_r \leq \lambda$, then elastic local buckling of the individual plate element occurs.

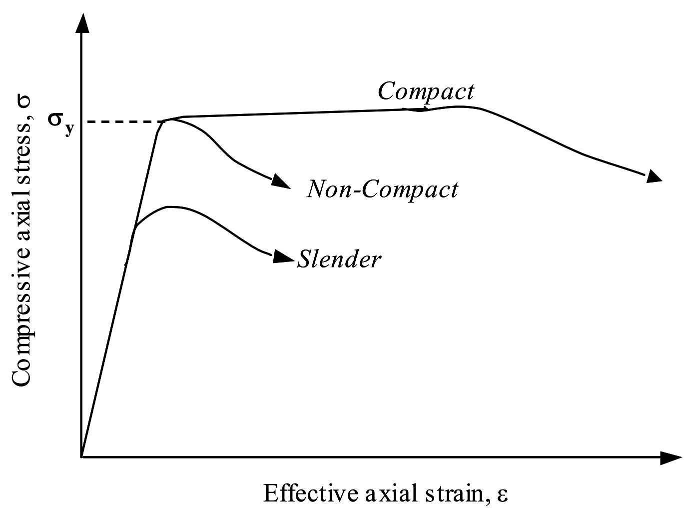


Figure 8. Stress-strain response of plates subjected to axial compression and local buckling.

- Thus, slender sections cannot develop M_p due to elastic local buckling. Non-compact sections can develop M_y but not M_p before local buckling occurs. Only compact sections can develop the plastic moment M_p.
- All rolled wide-flange shapes are <u>**compact**</u> with the following exceptions, which are non-compact.

- W40x174, W14x99, W14x90, W12x65, W10x12, W8x10, W6x15 (made from A992)

- The definition of λ and the values for λ_p and λ_r for the individual elements of various cross-sections are given in Table B5.1 and shown graphically on page 16.1-183. For example,

Section	**Plate element**	λ	λ_p	λ_r
Wide-flange	Flange	$b_f/2t_f$	$0.38\sqrt{E/F_y}$	$0.38\sqrt{E/F_L}$
	Web	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$
Channel	Flange	b_f/t_f	$0.38\sqrt{E/F_y}$	$0.38\sqrt{E/F_L}$
	Web	h/t_w	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$
Square or Rect. Box	Flange	(b-3t)/t	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$
	Web	(b-3t)/t	$3.76\sqrt{E/F_y}$	$5.70\sqrt{E/F_y}$

In CE405 we will design all beam sections to be compact from a local buckling standpoint

2.4 Lateral-Torsional Buckling

- The laterally unsupported length of a beam-member can undergo lateral-torsional buckling due to the applied flexural loading (bending moment).

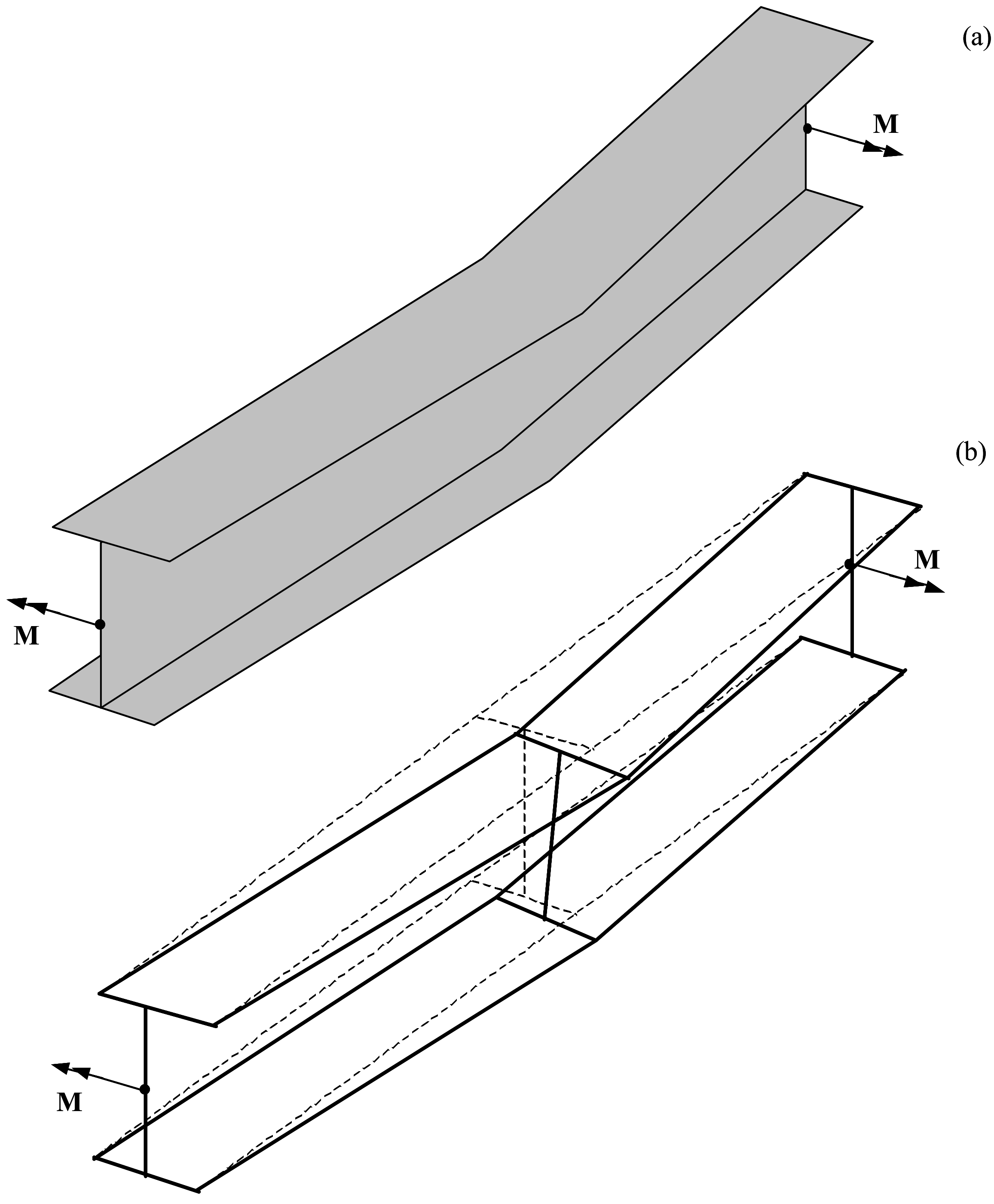


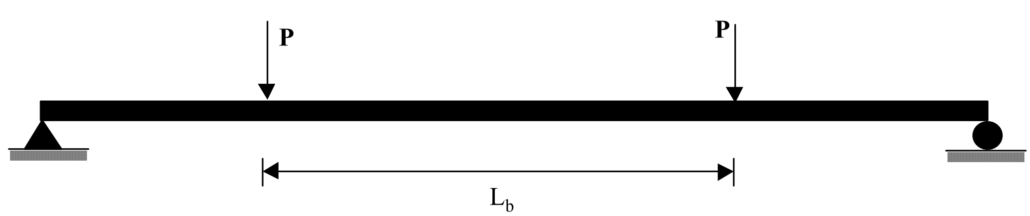
Figure 9. Lateral-torsional buckling of a wide-flange beam subjected to constant moment.

- Lateral-torsional buckling is fundamentally similar to the flexural buckling or flexural-torsional buckling of a column subjected to axial loading.
  - The similarity is that it is also a bifurcation-buckling type phenomenon.
  - The differences are that lateral-torsional buckling is caused by flexural loading (M), and the buckling deformations are coupled in the lateral and torsional directions.

  - There is one very important difference. For a column, the axial load causing buckling remains constant along the length. But, for a beam, usually the lateral-torsional buckling causing bending moment M(x) varies along the unbraced length.
  - The worst situation is for beams subjected to uniform bending moment along the unbraced length. Why?

2.4.1 Lateral-torsional buckling – Uniform bending moment

- Consider a beam that is simply-supported at the ends and subjected to four-point loading as shown below. The beam center-span is subjected to uniform bending moment M. Assume that lateral supports are provided at the load points.



- Laterally unsupported length = L_b.
- If the laterally unbraced length L_b is less than or equal to a plastic length L_p then lateral torsional buckling is not a problem and the beam will develop its plastic strength M_p.
- $L_p = 1.76\ r_y \times \sqrt{E/F_y}$ - for I members & channels (See Pg. **16.1**-33)

- If L_b is greater than L_p then lateral torsional buckling will occur and the moment capacity of the beam will be reduced below the plastic strength M_p as shown in Figure 10 below.

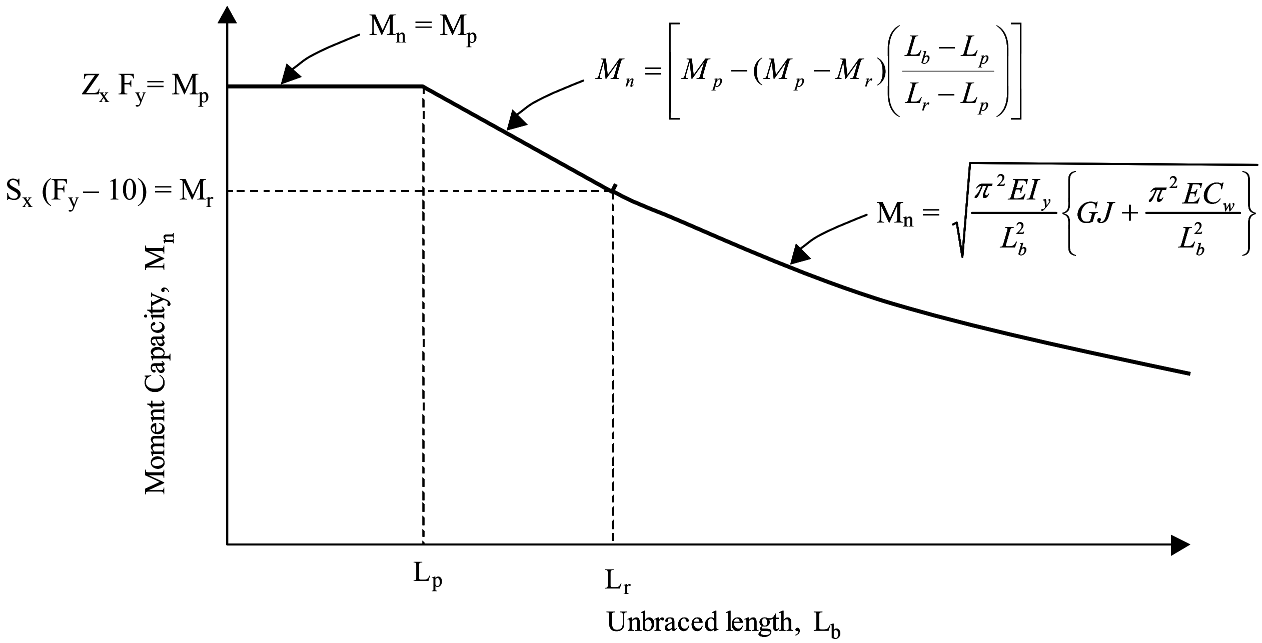


Figure 10. Moment capacity (M_n) versus unsupported length (L_b).

- As shown in Figure 10 above, the lateral-torsional buckling moment ($M_n = M_{cr}$) is a function of the laterally unbraced length L_b and can be calculated using the equation:

$$M_n = M_{cr} = \frac{\pi}{L_b}\sqrt{E \times I_y \times G \times J + \left(\frac{\pi \times E}{L_b}\right)^2 \times I_y \times C_w}$$

where, M_n = moment capacity

L_b = laterally unsupported length.

M_{cr} = critical lateral-torsional buckling moment.

E = 29000 ksi; G = 11,200 ksi

I_y = moment of inertia about minor or y-axis (in^4)

J = torsional constant (in^4) from the AISC manual pages _______________.

C_w = warping constant (in^6) from the AISC manual pages _______________.

- This equation is valid for **ELASTIC** lateral torsional buckling only (like the Euler equation). That is it will work only as long as the cross-section is elastic and no portion of the cross-section has yielded.
- As soon as any portion of the cross-section reaches the yield stress F_y, the elastic lateral torsional buckling equation cannot be used.
  - L_r is the unbraced length that corresponds to a lateral-torsional buckling moment $M_r = S_x\,(F_y - 10)$.
  - M_r will cause yielding of the cross-section due to residual stresses.
- When the unbraced length is less than L_r, then the elastic lateral torsional buckling equation cannot be used.
- When the unbraced length (L_b) is less than L_r but more than the plastic length L_p, then the lateral-torsional buckling M_n is given by the equation below:
  - If $L_p \le L_b \le L_r$, then $M_n = \left[M_p - (M_p - M_r)\left(\dfrac{L_b - L_p}{L_r - L_p}\right)\right]$
  - This is linear interpolation between (L_p, M_p) and (L_r, M_r)
  - See Figure 10 again.

2.4.2 Moment Capacity of beams subjected to non-uniform bending moments

- As mentioned previously, the case with uniform bending moment is worst for lateral torsional buckling.
- For cases with non-uniform bending moment, the lateral torsional buckling moment **is greater** than that for the case with uniform moment.
- The AISC specification says that:
  - The lateral torsional buckling moment for non-uniform bending moment case

    = $\mathbf{C_b}$ **x** lateral torsional buckling moment for uniform moment case.
- $\mathbf{C_b}$ is always greater than 1.0 for non-uniform bending moment.
  - $\mathbf{C_b}$ is equal to 1.0 for uniform bending moment.
  - Sometimes, if you cannot calculate or figure out $\mathbf{C_b}$, then it can be conservatively assumed as 1.0.
- $$C_b = \frac{12.5\, M_{max}}{2.5\, M_{max} + 3\, M_A + 4\, M_B + 3\, M_c}$$

  where, $\mathbf{M_{max}}$ = magnitude of maximum bending moment in $\mathbf{L_b}$

  $\mathbf{M_A}$ = magnitude of bending moment at quarter point of $\mathbf{L_b}$

  $\mathbf{M_B}$ = magnitude of bending moment at half point of $\mathbf{L_b}$

  $\mathbf{M_C}$ = magnitude of bending moment at three-quarter point of $\mathbf{L_b}$
- The moment capacity M_n for the case of non-uniform bending moment
  - $M_n = \mathbf{C_b}$ x {M_n for the case of uniform bending moment} $\leq \mathbf{M_p}$
  - Important to note that the increased moment capacity for the non-uniform moment case cannot possibly be more than $\mathbf{M_p}$.
  - Therefore, if the calculated values is greater than $\mathbf{M_p}$, then you have to reduce it to $\mathbf{M_p}$

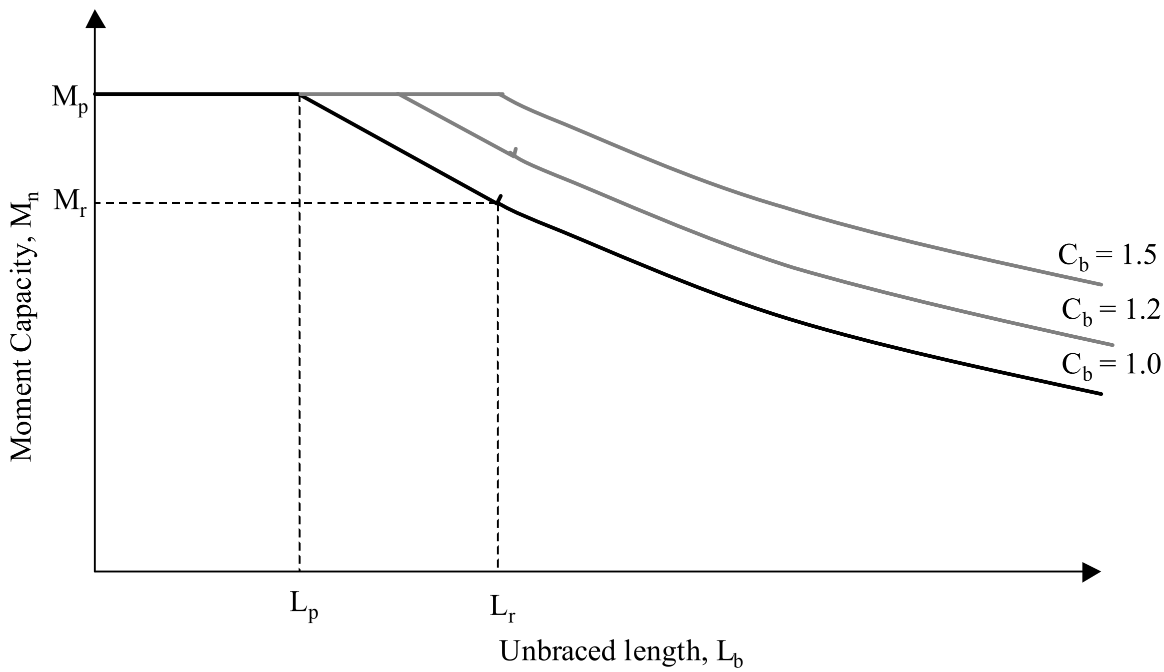
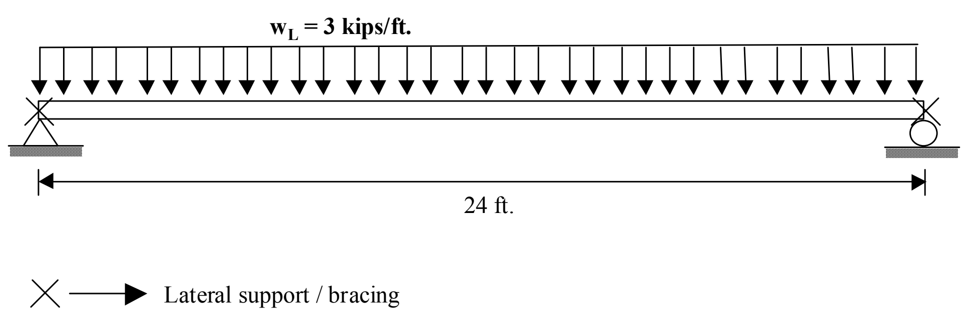


Figure 11. Moment capacity versus L_b for non-uniform moment case.

2.5 Beam Design

Example 2.4

Design the beam shown below. The unfactored uniformly distributed live load is equal to 3 kips/ft. There is no dead load. Lateral support is provided at the end reactions.



Step I. Calculate the factored loads assuming a reasonable self-weight.

Assume self-weight = w_{sw} = 100 lbs/ft.

Dead load = w_D = 0 + 0.1 = 0.1 kips/ft.

Live load = w_L = 3.0 kips/ft.

Ultimate load = w_u = 1.2 w_D + 1.6 w_L = 4.92 kips/ft.

Factored ultimate moment = M_u = w_u $L^2/8$ = 354.24 kip-ft.

Step II. Determine unsupported length L_b and C_b

There is only one unsupported span with L_b = 24 ft.

C_b = 1.14 for the parabolic bending moment diagram, See values of C_b shown in Figure.

Step III. Select a wide-flange shape

The moment capacity of the selected section $\phi_b M_n > M_u$ (Note $\phi_b = 0.9$)

$\phi_b M_n$ = moment capacity = C_b x ($\phi_b M_n$ for the case with **uniform moment**) $\leq \phi_b M_p$

- Pages __________________ in the AISC-LRFD manual, show the plots of $\phi_b M_n$-L_b for the case of uniform bending moment (C_b=1.0)
- Therefore, in order to select a section, calculate M_u/C_b and use it with L_b to find the first section with a **<u>solid line</u>** as shown in class.
- M_u/C_b = 354.24/1.14 = 310.74 kip-ft.
- Select W16 x 67 (50 ksi steel) with $\phi_b M_n$ =357 kip-ft. for L_b = 24 ft. and C_b =1.0
- For the case with C_b = 1.14,

  $\phi_b M_n$ = 1.14 x 357 = 406.7 kip-ft., **which <u>must</u> be $\leq \phi_b M_p$ = 491 kip-ft**.

  <u>OK!</u>

- *Thus, W16 x 67 made from 50 ksi steel with moment capacity equal to 406.7 kip-ft. for an unsupported length of 24 ft. is the designed section.*

Step IV. Check for local buckling.

$\lambda = b_f / 2t_f = 7.7$; Corresponding $\lambda_p = 0.38\ (E/Fy)^{0.5} = 9.192$

Therefore, $\lambda < \lambda_p$ - compact flange

$\lambda = h/t_w = 34.4$; Corresponding $\lambda_p = 3.76\ (E/F_y)^{0.5} = 90.5$

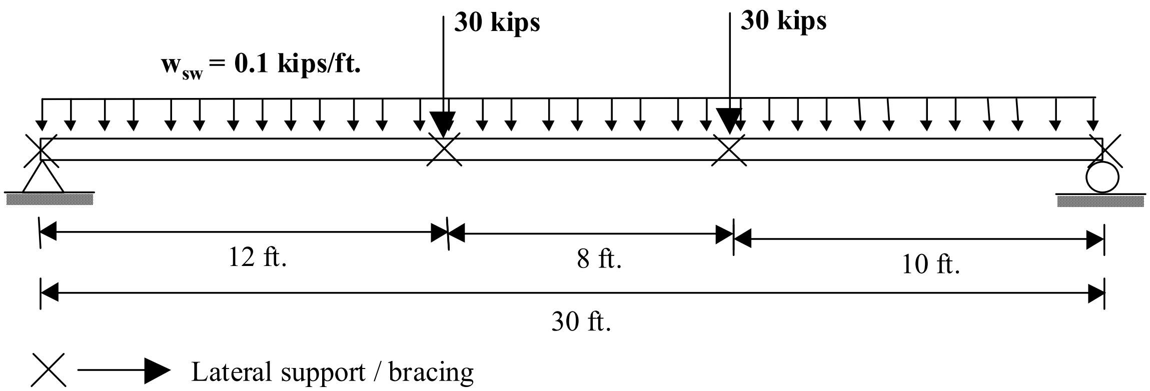
Therefore, $\lambda < \lambda_p$ - compact web

Compact section. - OK!

- *This example demonstrates the method for designing beams and accounting for $C_b > 1.0$*

Example 2.5

Design the beam shown below. The concentrated live loads acting on the beam are shown in the Figure. The beam is laterally supported at the load and reaction points.



Step I. Assume a self-weight and determine the factored design loads

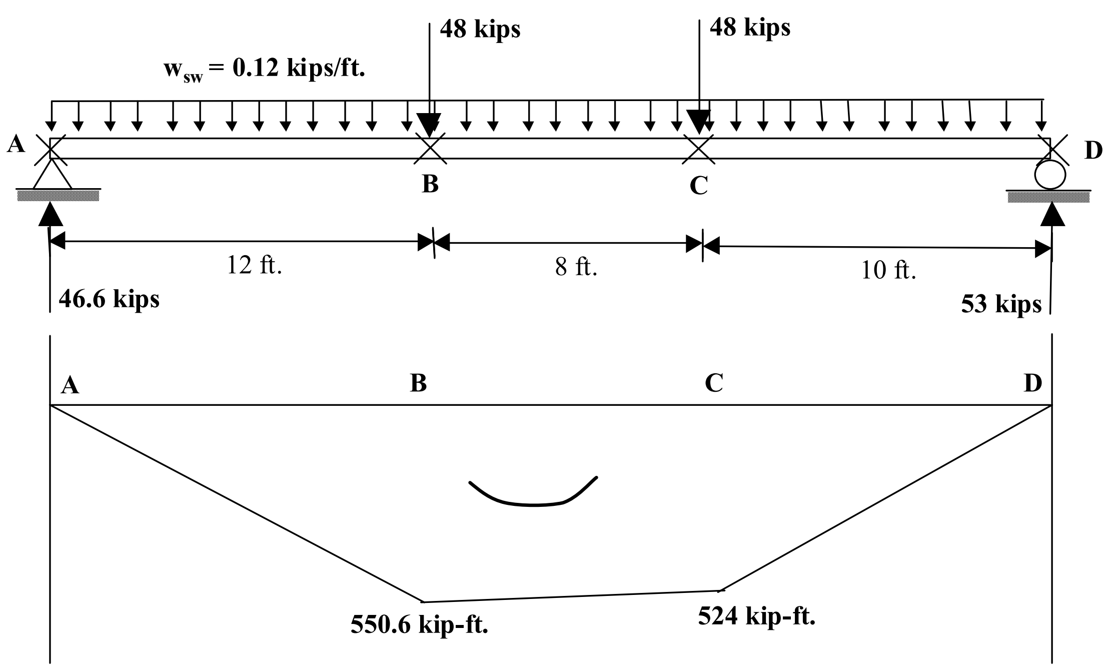
Let, w_{sw} = 100 lbs/ft. = 0.1 kips/ft.

P_L = 30 kips

$P_u = 1.6\ P_L$ = 48 kips

$w_u = 1.2 \text{ x } w_{sw}$ = 0.12 kips/ft.

The reactions and bending moment diagram for the beam are shown below.



Step II. Determine L_b, C_b, M_u, and M_u/C_b for all spans.

Span	L_b (ft.)	C_b	M_u (kip-ft.)	M_u/C_b (kip-ft.)
AB	12	1.67	550.6	329.7
BC	8	1.0 (assume)	550.6	550.6
CD	10	1.67	524.0	313.8

It is important to note that it is possible to have different L_b and C_b values for different laterally unsupported spans of the same beam.

Step III. Design the beam and check all laterally unsupported spans

Assume that **span BC** is the controlling span because it has the largest M_u/C_b although the corresponding L_b is the smallest.

From the AISC-LRFD manual select W21 x 68 made from 50 ksi steel (page _____)

Check the selected section for spans **AB, BC, and CD**

Span	L_b (ft.)	$\phi_b M_n$ for $C_b = 1.0$ *from _______*	C_b	$\phi_b M_n$ for C_b value *col. 3 x col. 4*	$\phi_b M_p$ *limit*
AB	12	507	1.67	846.7	600 kip-ft
BC	8	572	1.0	572.0	
CD	10	540	1.67	901.8	600 kip-ft.

Thus, for span AB, $\phi_b M_n$ = 600 kip-ft. > M_u - OK!

for span BC, $\phi_b M_n$ = 572.0 kip-ft. > M_u -OK!

For span CD, $\phi_b M_n$ = 600 kip-ft. > M_u -OK!

Step IV. Check for local buckling

$\lambda = b_f / 2t_f = 6.0$; Corresponding $\lambda_p = 0.38\ (E/Fy)^{0.5} = 9.192$

Therefore, $\lambda < \lambda_p$ - compact flange

$\lambda = h/t_w = 43.6$; Corresponding $\lambda_p = 3.76\ (E/F_y)^{0.5} = 90.55$

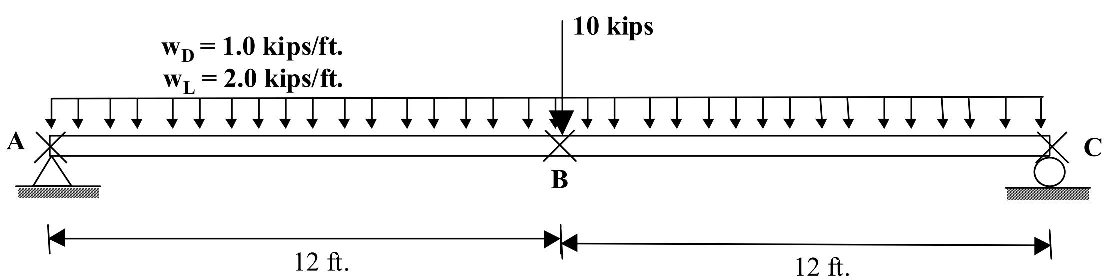
Therefore, $\lambda < \lambda_p$ - compact web

Compact section. - OK!

This example demonstrates the method for designing beams with several laterally unsupported spans with different L_b and C_b values.

Example 2.6

Design the simply-supported beam shown below. The uniformly distributed dead load is equal to 1 kips/ft. and the uniformly distributed live load is equal to 2 kips/ft. A concentrated live load equal to 10 kips acts at the mid-span. Lateral supports are provided at the end reactions and at the mid-span.



Step I. Assume the self-weight and calculate the factored design loads.

Let, w_{sw} = 100 lbs/ft. = 0.1 kips/ft.

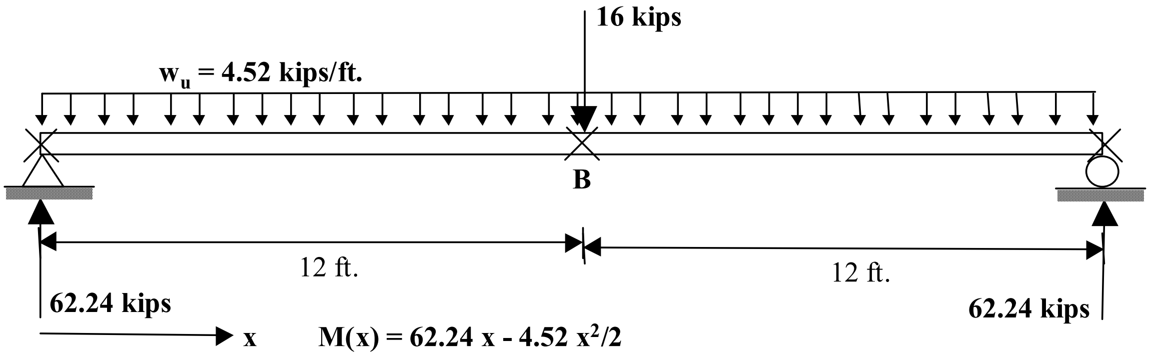
$w_D = 1 + 0.1 = 1.1$ kips/ft.

$w_L = 2.0$ kips/ft.

$w_u = 1.2\ w_D + 1.6\ w_L = 4.52$ kips/ft.

$P_u = 1.6 \times 10 = 16.0$ kips

The reactions and the bending moment diagram for the factored loads are shown below.



Step II. Calculate L_b and C_b for the laterally unsupported spans.

Since this is a symmetric problem, need to consider only span AB

L_b = 12 ft.; $C_b = \dfrac{12.5\, M_{max}}{2.5\, M_{max} + 3\, M_A + 4\, M_B + 3\, M_c}$

$M(x) = 62.24\,x - 4.52\,x^2/2$

Therefore,

$M_A = M(x = 3 \text{ ft.}) = 166.38$ kip-ft. - quarter-point along L_b = 12 ft.

$M_B = M(x = 6 \text{ ft.}) = 292.08$ kip-ft. - half-point along L_b = 12 ft.

$M_C = M(x = 9\text{ft.}) = 377.1$ kip-ft -three-quarter point along L_b= 12 ft.

$M_{max} = M(x = 12 \text{ ft.}) = 421.44$ kip-ft. - maximum moment along L_b =12ft.

Therefore, $C_b = 1.37$

Step III. Design the beam section

$M_u = M_{max} = 421.44$ kip-ft.

L_b = 12.0 ft.; C_b = 1.37

$M_u/C_b = 421.44/1.37 = 307.62$ kip-ft.

- Select W21 x 48 made from 50 ksi with $\phi_b M_n$ = 322 kip-ft. for L_b = 12.0 ft. and C_b =1.0
- For C_b = 1.37, $\phi_b M_n$ = 441.44 k-ft., but must be < or = $\phi_b M_p$ = 398 k-ft.

- Therefore, for C_b =1.37, $\phi_b M_n$ = <u>398 k-ft.</u> < M_u

Step IV. Redesign the section

- Select the next section with greater capacity than W21 x 48
- Select W18 x 55 with $\phi_b M_n$ = 345 k-ft. for L_b = 12 ft. and C_b = 1.0

  For C_b = 1.37, $\phi_b M_n$ = 345 x 1.37 = 472.65 k-ft. but must be ≤ $\phi_b M_p$ = 420 k-ft.

  Therefore, for C_b = 1.37, $\phi_b M_n$ = 420 k-ft., which is < M_u (421.44 k-ft), (**NOT OK!**)

- Select W 21 x 55 with $\phi_b M_n$ = 388 k-ft. for L_b = 12 ft. and C_b = 1.0

  For C_b 1.37, $\phi_b M_n$ = 388 x 1.37 = 531.56 k-ft., but must be ≤ $\phi_b M_p$ = 473 k-ft.

  Therefore, for C_b = 1.37, $\phi_b M_n$ = 473 k-ft, which is > M_u (421.44 k-ft). (**<u>OK!</u>**)

Step V. Check for local buckling.

$\lambda = b_f / 2t_f$ = <u>7.87</u>; Corresponding $\lambda_p = 0.38\,(E/F_y)^{0.5} = 9.192$

Therefore, $\lambda < \lambda_p$ - compact flange

$\lambda = h/t_w$ = <u>50.0</u>; Corresponding $\lambda_p = 3.76\,(E/F_y)^{0.5} = 90.55$

Therefore, $\lambda < \lambda_p$ - compact web

Compact section. **- OK!**

This example demonstrates the calculation of C_b and the iterative design method.

CHAPTER 3. COMPRESSION MEMBER DESIGN

3.1 INTRODUCTORY CONCEPTS

- Compression Members: Structural elements that are subjected to axial compressive forces only are called *columns*. Columns are subjected to axial loads thru the centroid.
- Stress: The stress in the column cross-section can be calculated as

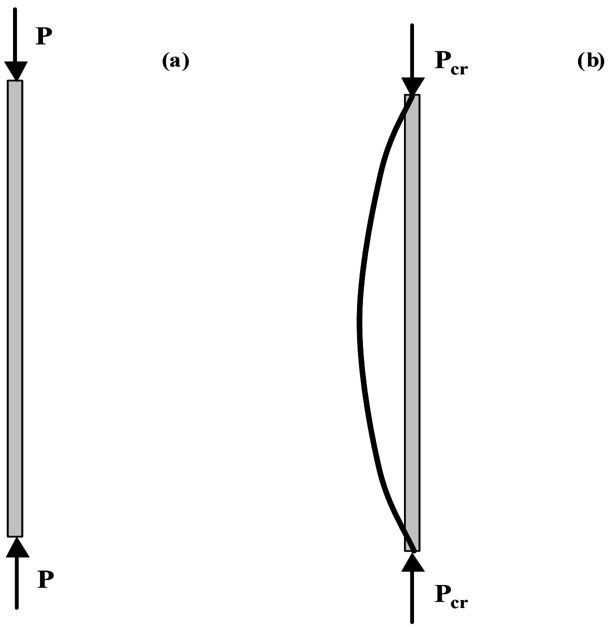
$$f = \frac{P}{A} \tag{2.1}$$

where, f is assumed to be uniform over the entire cross-section.

- This ideal state is never reached. The stress-state will be non-uniform due to:
  - Accidental eccentricity of loading with respect to the centroid
  - Member out-of –straightness (crookedness), or
  - Residual stresses in the member cross-section due to fabrication processes.
- Accidental eccentricity and member out-of-straightness can cause bending moments in the member. However, these are secondary and are usually ignored.
- Bending moments cannot be neglected if they are acting on the member. Members with axial compression and bending moment are called *beam-columns*.

3.2 COLUMN BUCKLING

- Consider a long slender compression member. If an axial load P is applied and increased slowly, it will ultimately reach a value P_{cr} that will cause buckling of the column. P_{cr} is called the critical buckling load of the column.



What is buckling?

Buckling occurs when a straight column subjected to axial compression suddenly undergoes bending as shown in the Figure 1(b). Buckling is identified as a failure limit-state for columns.

Figure 1. Buckling of axially loaded compression members

- The critical buckling load P_{cr} for columns is theoretically given by Equation (3.1)

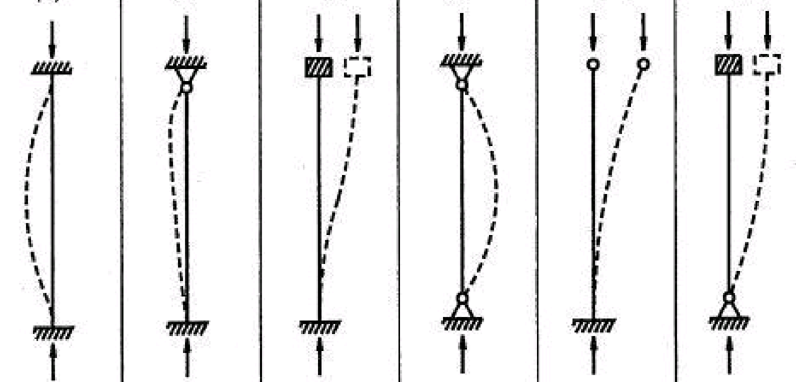
$$P_{cr} = \frac{\pi^2 E\, I}{(K\; L)^2} \tag{3.1}$$

where, I = moment of inertia about axis of buckling

K = effective length factor based on end boundary conditions

- Effective length factors are given on page 16.1-189 of the AISC manual.

Table C-C2.1
K Values for Columns



Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K Value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal condition are approximated	0.65	0.80	1.2	1.0	2.10	2.0

End Condition code

Rotation fixed and translation fixed
Rotation free and translation fixed
Rotation fixed and translation free
Rotation free and translation free

- In examples, homeworks, and exams please state clearly whether you are using the theoretical value of *K* or the recommended design values.

<u>EXAMPLE 3.1</u> Determine the buckling strength of a W 12 x 50 column. Its length is 20 ft. For major axis buckling, it is pinned at both ends. For minor buckling, is it pinned at one end and fixed at the other end.

<u>Solution</u>

Step I. Visualize the problem

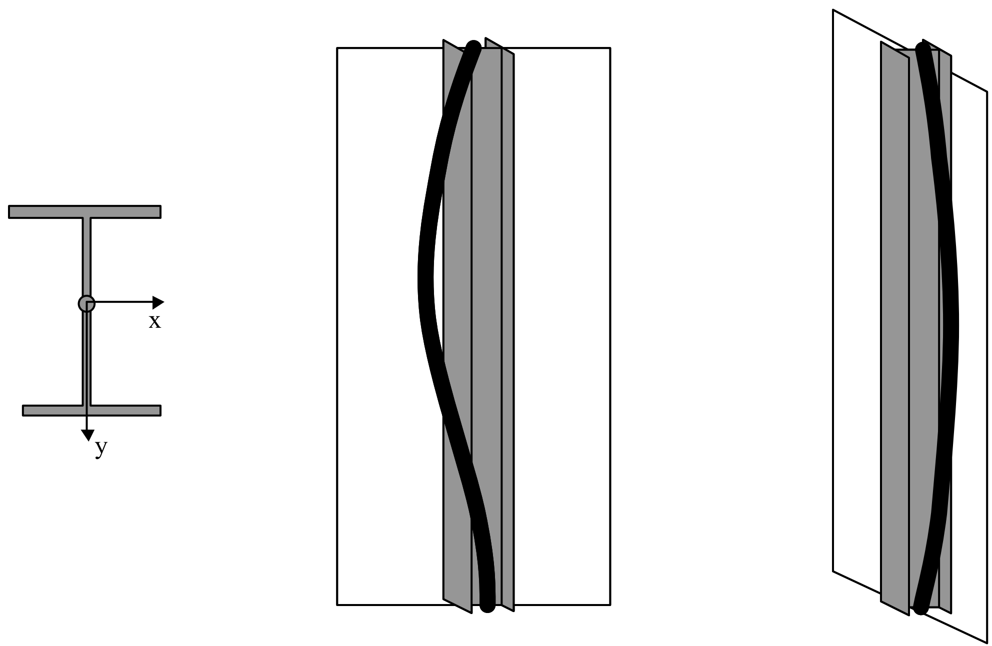


Figure 2. (a) Cross-section; (b) major-axis buckling; (c) minor-axis buckling

- For the W12 x 50 (or any wide flange section), x is the major axis and y is the minor axis. Major axis means axis about which it has greater moment of inertia ($I_x > I_y$)

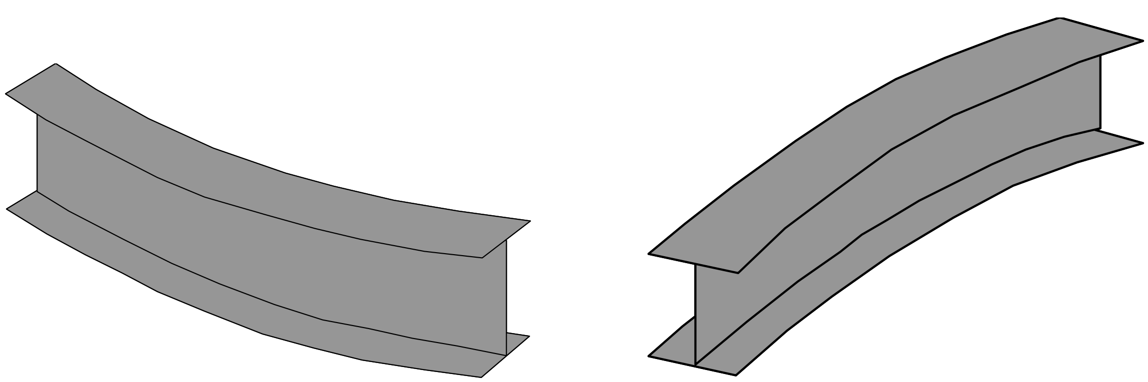


Figure 3. (a) Major axis buckling; (b) minor axis buckling

Step II. Determine the effective lengths

- According to Table C-C2.1 of the AISC Manual (see page 16.1 - 189):
  - For pin-pin end conditions about the minor axis

    K_y = 1.0 (theoretical value); and K_y = 1.0 (recommended design value)
  - For pin-fix end conditions about the major axis

    K_x = 0.7 (theoretical value); and K_x = 0.8 (recommended design value)
- According to the problem statement, the unsupported length for buckling about the major (x) axis = L_x = 20 ft.
- The unsupported length for buckling about the minor (y) axis = L_y = 20 ft.
- Effective length for major (x) axis buckling = $K_x L_x$ = 0.8 x 20 = 16 ft. = 192 in.
- Effective length for minor (y) axis buckling = $K_y L_y$ = 1.0 x 20 = 20 ft. = 240 in.

Step III. Determine the relevant section properties

- For W12 x 50: elastic modulus = E = 29000 ksi (constant for all steels)
- For W12 x 50: $I_x = 391\ in^4$. $I_y = 56.3\ in^4$ (see page 1-21 of the AISC manual)

Step IV. Calculate the buckling strength

- Critical load for buckling about x - axis = $P_{cr\text{-}x} = \dfrac{\pi^2 E I_x}{(K_x\ L_x)^2} = \dfrac{\pi^2 \times 29000 \times 391}{(192)^2}$

$$P_{cr\text{-}x} = 3035.8 \text{ kips}$$

- Critical load for buckling about y-axis = $P_{cr\text{-}y} = \dfrac{\pi^2 E I_y}{(K_y\ L_y)^2} = \dfrac{\pi^2 \times 29000 \times 56.3}{(240)^2}$

$$P_{cr\text{-}y} = 279.8 \text{ kips}$$

- Buckling strength of the column = smaller ($P_{cr\text{-}x}$, $P_{cr\text{-}y}$) = <u>P_{cr} = 279.8 kips</u>

  <u>Minor (y) axis buckling governs.</u>

- ***Notes:***

- *Minor axis buckling usually governs for all doubly symmetric cross-sections. However, for some cases, major (x) axis buckling can govern.*
- *Note that the steel yield stress was irrelevant for calculating this buckling strength.*

3.3 INELASTIC COLUMN BUCKLING

- Let us consider the previous example. According to our calculations P_{cr} = 279.8 kips. This P_{cr} will cause a uniform stress $f = P_{cr}/A$ in the cross-section
- For W12 x 50, A = 14.6 in^2. Therefore, for P_{cr} = 279.8 kips; f = 19.16 ksi

  The calculated value of f is within the elastic range for a 50 ksi yield stress material.
- However, if the unsupported length was only 10 ft., $P_{cr} = \dfrac{\pi^2 E\, I_y}{(K_y\ L_y)^2}$ would be calculated as 1119 kips, and f = 76.6 kips.
- This value of f is ridiculous because the material will yield at 50 ksi and never develop f = 76.6 kips. <u>The member would yield before buckling.</u>
- **Equation (3.1) is valid only when the material everywhere in the cross-section is in the elastic region. If the material goes inelastic then Equation (3.1) becomes useless and cannot be used.**
- What happens in the inelastic range?

  Several other problems appear in the inelastic range.
  - The member out-of-straightness has a significant influence on the buckling strength in the inelastic region. It must be accounted for.

- The residual stresses in the member due to the fabrication process causes yielding in the cross-section much before the uniform stress f reaches the yield stress F_y.
- The shape of the cross-section (W, C, etc.) also influences the buckling strength.
- In the inelastic range, the steel material can undergo strain hardening.

All of these are very advanced concepts and beyond the scope of CE405. You are welcome to CE805 to develop a better understanding of these issues.

- So, what should we do? We will directly look at the AISC Specifications for the strength of compression members, i.e., Chapter E (page 16.1-27 of the AISC manual).

3.4 AISC SPECIFICATIONS FOR COLUMN STRENGTH

- The AISC specifications for column design are based on several years of research.
- These specifications account for the elastic and inelastic buckling of columns including all issues (member crookedness, residual stresses, accidental eccentricity etc.) mentioned above.
- The specification presented here (AISC Spec E2) will work for all doubly symmetric cross-sections and channel sections.
- The design strength of columns for the flexural buckling limit state is equal to $\boldsymbol{\phi_c P_n}$

Where, $\phi_c = 0.85$ (Resistance factor for compression members)

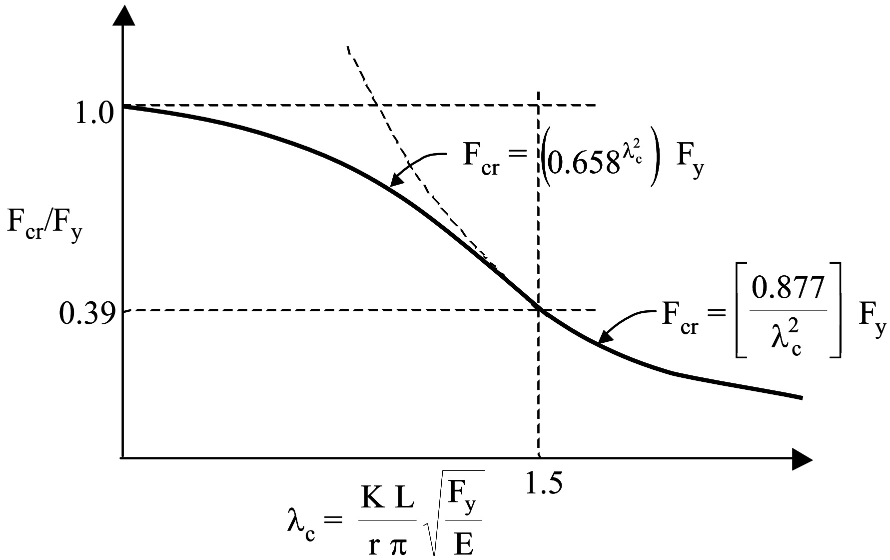
$$P_n = A_g F_{cr} \tag{3.2}$$

- For $\lambda_c \leq 1.5$ $\quad F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y$ (3.3)
- For $\lambda_c > 1.5$ $\quad F_{cr} = \left[\dfrac{0.877}{\lambda_c^2}\right] F_y$ (3.4)

Where, $\lambda_c = \dfrac{K\,L}{r\,\pi}\sqrt{\dfrac{F_y}{E}}$ (3.5)

A_g = gross member area; K = effective length factor

L = unbraced length of the member; r = governing radius of gyration



- Note that the original Euler buckling equation is $P_{cr} = \dfrac{\pi^2 E I}{(K\,L)^2}$

$$\therefore F_{cr} = \frac{P_{cr}}{A_g} = \frac{\pi^2 E}{(K\,L)^2} \times \frac{I}{A_g} = \frac{\pi^2 E}{(K\,L)^2} \times r^2 = \frac{\pi^2 E}{\left(\frac{K\,L}{r}\right)^2}$$

$$\therefore \frac{F_{cr}}{F_y} = \frac{\pi^2 E}{\left(\frac{K\,L}{r}\right)^2 \times F_y} = \frac{1}{\left(\frac{K\,L}{r\,\pi} \times \sqrt{\frac{F_y}{E}}\right)^2} = \frac{1}{\lambda_c^2}$$

$$\therefore F_{cr} = F_y \times \frac{1}{\lambda_c^2}$$

- Note that the AISC equation for $\boldsymbol{\lambda_c < 1.5}$ is $F_{cr} = F_y \times \dfrac{0.877}{\lambda_c^2}$
  - The 0.877 factor tries to account for initial crookedness.

- For a given column section:
  - Calculate I, A_g, r
  - Determine effective length *K L* based on end boundary conditions.
  - Calculate λ_c
  - If λ_c is greater than 1.5, *elastic buckling* occurs and use Equation (3.4)

- If λ_c is less than or equal to 1.5, *inelastic buckling* occurs and use Equation (3.3)

- Note that the column can develop its yield strength F_y as λ_c approaches zero.

3.5 COLUMN STRENGTH

- In order to simplify calculations, the AISC specification includes Tables.
  - Table 3-36 on page **16.1**-143 shows KL/r vs. $\phi_c F_{cr}$ for steels with F_y = 36 ksi.
  - You can calculate KL/r for the column, then read the value of $\phi_c F_{cr}$ from this table
  - The column strength will be equal to $\phi_c F_{cr}$ x A_g
  - Table 3-50 on page **16.1**-145 shows KL/r vs. $\phi_c F_{cr}$ for steels with F_y = 50 ksi.
- In order to simplify calculations, the AISC specification includes more Tables.
  - Table 4 on page **16.1**-147 shows λ_c vs. $\phi_c F_{cr}/F_y$ for all steels with any F_y.
  - You can calculate λ_c for the column, the read the value of $\phi_c F_{cr}/F_y$
  - The column strength will be equal to $\phi_c F_{cr}/F_y$ x (A_g x F_y)

<u>**EXAMPLE 3.2**</u> Calculate the design strength of W14 x 74 with length of 20 ft. and pinned ends. A36 steel is used.

<u>Solution</u>

- Step I. Calculate the effective length and slenderness ratio for the problem

  $K_x = K_y = 1.0$

  $L_x = L_y = 240$ in.

  Major axis slenderness ratio = $K_x L_x / r_x = 240/6.04 = 39.735$

  Minor axis slenderness ratio = $K_y L_y / r_y = 240/2.48 = 96.77$

- Step II. Calculate the buckling strength for governing slenderness ratio

The governing slenderness ratio is the larger of (K_xL_x/r_x, K_yL_y/r_y)

K_yL_y/r_y is larger and the governing slenderness ratio; $\lambda_c = \frac{K_y\ L_y}{r_y\ \pi}\sqrt{\frac{F_y}{E}} = 1.085$

$\lambda_c < 1.5$; Therefore, $F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y$

Therefore, $F_{cr} = 21.99$ ksi

Design column strength = $\phi_c P_n = 0.85\ (A_g\ F_{cr}) = 0.85\ (21.8\ in^2 \times 21.99\ ksi) = 408$ kips

Design strength of column = 408 kips

- Check calculated values with Table 3-36. For KL/r = 97, $\phi_c F_{cr} = 18.7$ ksi
- Check calculated values with Table 4. For $\lambda_c = 1.08$, $\phi_c F_{cr} = 0.521$

3.6 LOCAL BUCKLING LIMIT STATE

- The AISC specifications for column strength assume that column buckling is the governing limit state. However, if the column section is made of thin (slender) plate elements, then failure can occur due to *local buckling* of the flanges or the webs.

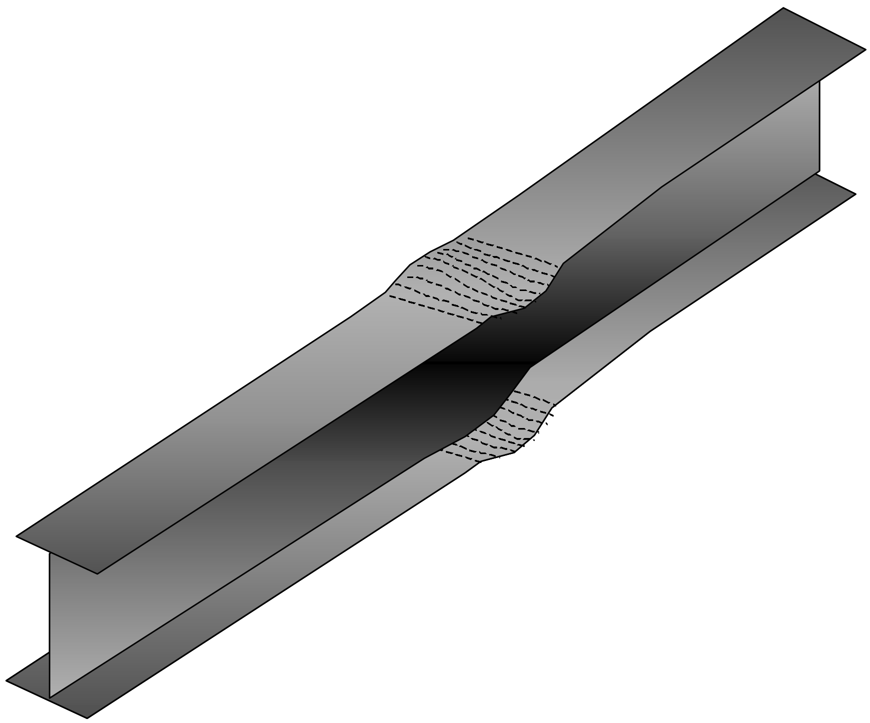


Figure 4. Local buckling of columns

- If *local buckling* of the individual plate elements occurs, then the column may not be able to develop its buckling strength.
- Therefore, the local buckling limit state <u>must be prevented</u> from controlling the column strength.
- Local buckling depends on the slenderness (width-to-thickness *b/t* ratio) of the plate element and the yield stress (F_y) of the material.
- Each plate element must be stocky enough, i.e., have a *b/t* ratio that prevents local buckling from governing the column strength.

- The AISC specification B5 provides the slenderness (b/t) limits that the individual plate elements must satisfy so that *local buckling* does not control.
- The AISC specification provides two slenderness limits (λ_p and λ_r) for the local buckling of plate elements.

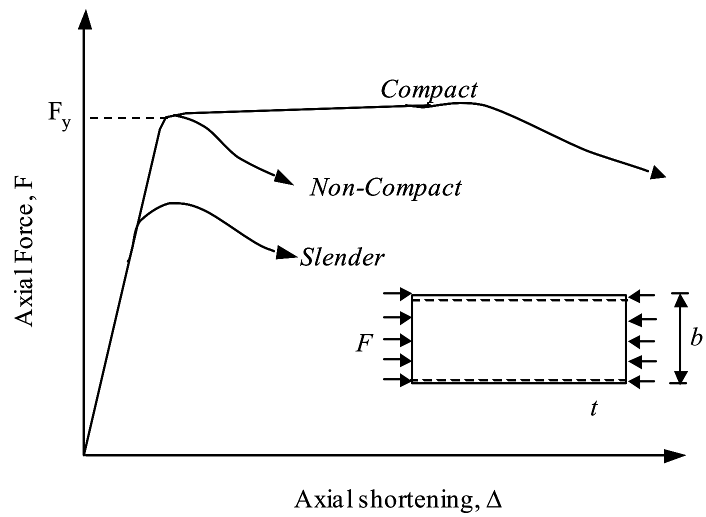


Figure 5. Local buckling behavior and classification of plate elements

- If the slenderness ratio (b/t) of the plate element is greater than λ_r then it *is slender*. It will locally buckle in the elastic range *before* reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_r but greater than λ_p, then it is *non-compact*. It will locally buckle *immediately* after reaching F_y
- If the slenderness ratio (b/t) of the plate element is less than λ_p, then the element is *compact*. It will locally buckle *much after* reaching F_y

- If all the plate elements of a cross-section are compact, then the section is *compact*.
  - If any one plate element is non-compact, then the cross-section is non-compact
  - If any one plate element is slender, then the cross-section is slender.
- The slenderness limits λ_p and λ_r for various plate elements with different boundary conditions are given in Table B5.1 on pages **16.1**-14 and **16.1**-15 of the AISC Spec.

- Note that the slenderness limits (λ_p and λ_r) and the definition of plate slenderness (b/t) ratio depend upon the boundary conditions for the plate.
  - If the plate is supported along *two edges* parallel to the direction of compression force, then it is a *stiffened* element. For example, the webs of W shapes
  - If the plate is supported along only *one edge* parallel to the direction of the compression force, then it is an *unstiffened* element. Ex., the flanges of W shapes.

- The local buckling limit state can be prevented from controlling the column strength by using sections that are non-compact
  - If all the elements of the cross-section have calculated slenderness (b/t) ratio less than λ_r, then the local buckling limit state will not control.
  - For the definitions of b/t, λ_p, λ_r for various situations see Table B5.1 and Spec B5.

EXAMPLE 3.3 Determine the local buckling slenderness limits and evaluate the W14 x 74 section used in Example 3.2. Does local buckling limit the column strength?

Solution

- Step I. Calculate the slenderness limits

  See Table B5.1 on page 16.1 – 14.

  - For the flanges of I-shape sections in pure compression

$$\lambda_r = 0.56 \text{ x } \sqrt{\frac{E}{F_y}} = 0.56 \text{ x } \sqrt{\frac{29000}{36}} = 15.9$$

  - For the webs of I-shapes section in pure compression

$$\lambda_r = 0.56 \text{ x } \sqrt{\frac{E}{F_y}} = 0.56 \text{ x } \sqrt{\frac{29000}{36}} = 15.9$$

$$\lambda_r = 1.49 \text{ x } \sqrt{\frac{E}{F_y}} = 1.49 \text{ x } \sqrt{\frac{29000}{36}} = 42.3$$

- Step II. Calculate the slenderness ratios for the flanges and webs of W14 x 74
  - For the flanges of I-shape member, $b = b_f/2$ = flange width / 2

    Therefore, $b/t = b_f/2t_f$. (See pg. **16.1**-12 of AISC)

    For W 14 x 74, $b_f/2t_f = 6.41$ (See Page 1-19 in AISC)
  - For the webs of I shaped member, b = h

    h is the clear distance between flanges less the fillet / corner radius of each flange

    For W14 x 74, $h/t_w = 25.4$ (See Page 1-19 in AISC)
- Step III. Make the comparisons and comment

  For the flanges, $b/t < \lambda_r$. Therefore, the flange is non-compact

  For the webs, $h/t_w < \lambda_r$. Therefore the web is non-compact

  Therefore, the section is <u>compact</u>

  Therefore, local buckling will not limit the column strength.

3.7 COLUMN DESIGN

- The AISC manual has tables for column strength. See page **4**-21 onwards.
- For wide flange sections, *the column buckling strength* ($\phi_c P_n$) *is tabulated with respect to the effective length about the minor axis $K_y L_y$* in Table 4-2.
  - The table takes the $K_y L_y$ value for a section, and <u>internally</u> calculates the $K_y L_y / r_y$, then $\lambda_c = \frac{K_y\ L_y}{r_y\ \pi}\sqrt{\frac{F_y}{E}}$; and then the *tabulated* column strength using either Equation E2-2 or E2-3 of the specification.

- If you want to use the Table 4-2 for calculating the column strength for buckling about *the major axis*, then do the following:
  - Take the major axis K_xL_x value. Calculate an equivalent $(KL)_{eq} = \dfrac{K_x L_x}{r_x / r_y}$
  - Use the calculated $(KL)_{eq}$ value to find $(\phi_c P_n)$ the column strength for buckling about the *major axis* from Table (4-2)
- For example, consider a W14 x 74 column with K_yL_y = 20 ft. and K_xL_x = 25 ft.
  - Material has yield stress = 50 ksi (<u>always</u> in Table 4-2).
  - See Table 4-2, for K_yL_y = 20 ft., $\phi_c P_n$ = 467 kips (minor axis buckling strength)
  - r_x/r_y for W14x74 = 2.44 from Table 4-2 (see page 4-23 of AISC).
  - For K_xL_x = 25 ft., $(KL)_{eq}$ = 25/2.44 = 10.25 ft.
  - For $(KL)_{eq}$ = 10.25 ft., $\phi_c P_n$ = 774 kips (major axis buckling strength)
  - If calculated value of $(KL)_{eq} < K_yL_y$ then minor axis buckling will govern.

<u>EXAMPLE 3.4</u> Determine the design strength of an ASTM A992 W14 x 132 that is part of a braced frame. Assume that the physical length L = 30 ft., the ends are pinned and the column is braced at the ends only for the X-X axis and braced at the ends and mid-height for the Y-Y axis.

<u>Solution</u>

- <u>Step I.</u> Calculate the *effective lengths*.

  For W14 x 132: r_x = 6.28 in; r_y = 3.76 in; A_g =38.8 in^2

  K_x = 1.0 and K_y = 1.0

  L_x = 30 ft. and L_y = 15 ft.

  K_xL_x = 30 ft. and K_yL_y = 15 ft.

- <u>Step II.</u> Determine the governing slenderness ratio

K_xL_x/r_x = 30 x 12 in./6.28 in.= 57.32

K_yL_y/r_y = 15 x 12 in./3.76 in. = 47.87

The larger slenderness ratio, therefore, buckling about the major axis will govern the column strength.

- Step III. Calculate the column strength

K_xL_x = 30 ft. Therefore, $(KL)_{eq} = \dfrac{K_xL_x}{r_x/r_y} = \dfrac{30}{6.28/3.76}$ = 17.96 ft.

From Table 4-2, for $(KL)_{eq}$ = 18.0 ft. $\phi_c P_n$ = 1300 kips (design column strength)

- Step IV. Check the local buckling limits

For the flanges, $b_f/2t_f$ = 7.15 < $\lambda_r = 0.56 \times \sqrt{\dfrac{E}{F_y}}$ = 13.5

For the web, h/t_w = 17.7 < $\lambda_r = 1.49 \times \sqrt{\dfrac{E}{F_y}}$ = 35.9

Therefore, the section is non-compact. OK.

EXAMPLE 3.5 A compression member is subjected to service loads of 165 kips dead load and 535 kips of live load. The member is 26 ft. long and pinned at each end. Use A992 (50 ksi) steel and select a W shape

Solution

- Calculate the factored design load P_u

$P_u = 1.2\ P_D + 1.6\ P_L$ = 1.2 x 165 + 1.6 x 535 = 1054 kips

- Select a W shape from the AISC manual Tables

For K_yL_y = 26 ft. and required strength = 1054 kips

- Select W14 x 145 from page 4-22. It has $\phi_c P_n$ = 1160 kips

- Select W12 x 170 from page 4-24. It has $\phi_c P_n$ = 1070 kips
- No no W10 will work. See Page 4-26
- W14 x 145 is the lightest.

- Note that column sections are usually W12 or W14. Usually sections bigger than W14 are usually not used as columns.

3.8 EFFECTIVE LENGTH OF COLUMNS IN FRAMES

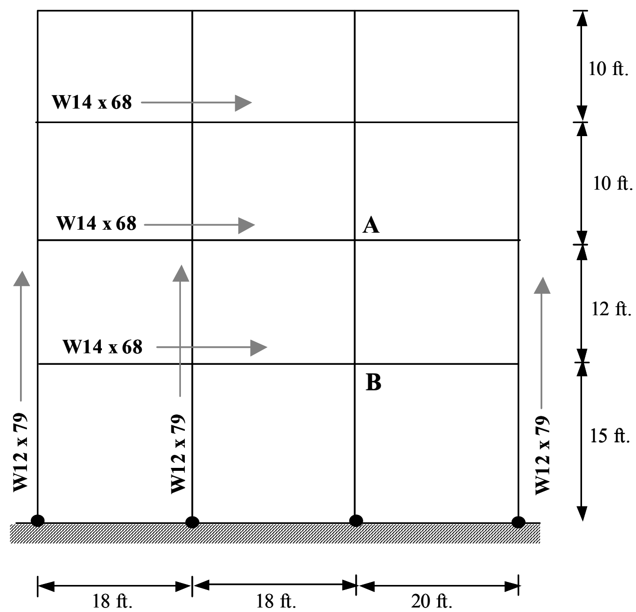
- So far, we have looked at the buckling strength of individual columns. These columns had various boundary conditions at the ends, but they were not connected to other members with moment (fix) connections.
- The effective length factor K for the buckling of an individual column can be obtained for the appropriate end conditions from Table C-C2.1 of the AISC Manual .
- However, when these individual columns are part of a frame, their ends are connected to other members (beams etc.).
  - Their effective length factor K will depend on the restraint offered by the other members connected at the ends.
  - Therefore, the effective length factor K will depend on the relative rigidity (stiffness) of the members connected at the ends.

The effective length factor for columns in frames must be calculated as follows:

- First, you have to determine whether the column is part of a braced frame or an unbraced (moment resisting) frame.
  - If the column is part of a braced frame then its effective length factor $0 < K \leq 1$
  - If the column is part of an unbraced frame then $1 < K \leq \infty$

- Then, you have to determine the relative rigidity factor G for both ends of the column
  - G is defined as the ratio of the summation of the rigidity (EI/L) of all columns coming together at an end to the summation of the rigidity (EI/L) of all beams coming together at the same end.
  - $G = \dfrac{\sum \dfrac{E\,I_c}{L_c}}{\sum \dfrac{E\,I_b}{L_b}}$ - It must be calculated for both ends of the column.
- Then, you can determine the effective length factor K for the column using the calculated value of G at both ends, i.e., G_A and G_B and the appropriate alignment chart
- There are two alignment charts provided by the AISC manual,
  - One is for columns in braced (sidesway inhibited) frames. See Figure C-C2.2a on page 16.1-191 of the AISC manual. $0 < K \leq 1$
  - The second is for columns in unbraced (sidesway uninhibited) frames. See Figure C-C2.2b on page 16.1-192 of the AISC manual. $1 < K \leq \infty$
  - The procedure for calculating G is the same for both cases.

<u>EXAMPLE 3.6</u> Calculate the effective length factor for the **W12 x 53** column AB of the frame shown below. Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame. Assume that the columns are braced at each story level for out-of-plane buckling. Assume that the same column section is used for the stories above and below.



Step I. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (sidesway uninhibited) frame.
- $L_x = L_y = 12$ ft.
- $K_y = 1.0$
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.

Step II - Calculate K_x

- I_{xx} of W 12 x 53 = 425 in^4 $\qquad$ I_{xx} of W14x68 = 753

- $$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{10 \times 12} + \frac{425}{12 \times 12}}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{6.493}{6.360} = 1.021$$

- $$G_B = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{12 \times 12} + \frac{425}{15 \times 12}}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{5.3125}{6.360} = 0.835$$

- Using G_A and G_B: $K_x = 1.3$ - from Alignment Chart on Page 3-6

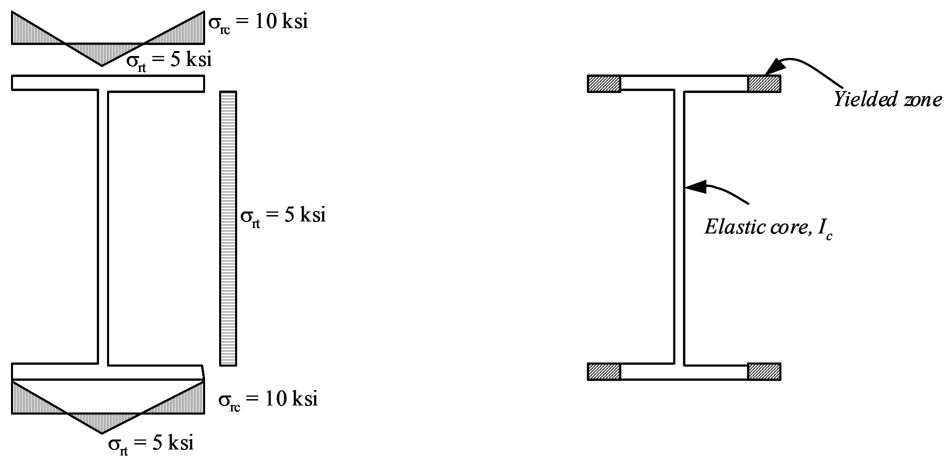
Step III – Design strength of the column

- $K_yL_y = 1.0 \times 12 = 12$ ft.
- $K_x L_x = 1.3 \times 12 = 15.6$ ft.
  - r_x / r_y for W12x53 = 2.11
  - $(KL)_{eq} = 15.6 / 2.11 = 7.4$ ft.
- $K_yL_y > (KL)_{eq}$
- Therefore, y-axis buckling governs. Therefore $\phi_c P_n = 518$ kips

3.8.1 Inelastic Stiffness Reduction Factor – Modification

- This concept for calculating the effective length of columns in frames was widely accepted for many years.
- Over the past few years, a lot of modifications have been proposed to this method due to its several assumptions and limitation. Most of these modifications have not yet been accepted in to the AISC provisions.
- One of the accepted modifications is the inelastic stiffness reduction factor. As presented earlier, G is a measure of the *relative flexural rigidity* of the columns (EI_c/L_c) with respect to the beams (EI_b/L_b)

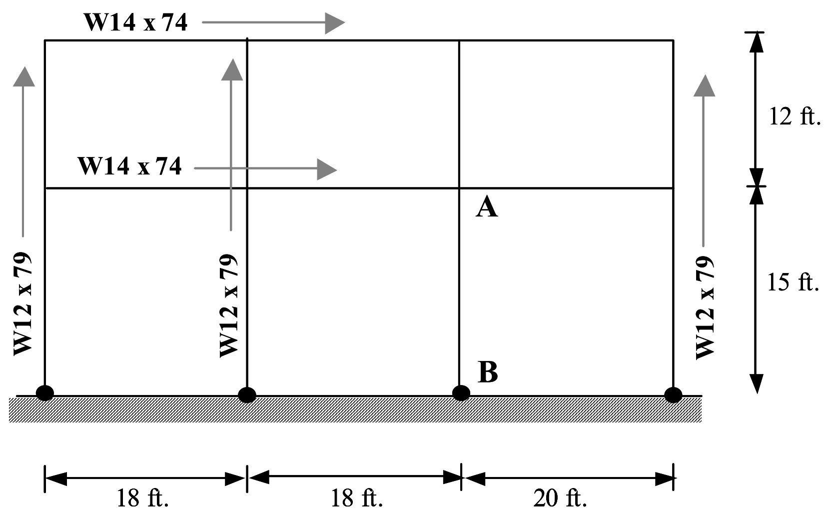
- However, if column buckling were to occur in the inelastic range ($\lambda_c < 1.5$), then the flexural rigidity of the column will be reduced because I_c will be the moment of inertia of only the elastic core of the entire cross-section. See figure below



(a) Initial state – residual stress (b) Partially yielded state at buckling

- The beams will have greater flexural rigidity when compared with the reduced rigidity (EI_c) of the inelastic columns. As a result, the beams will be able to restrain the columns better, which is good for column design.
- This effect is incorporated in to the AISC column design method through the use of Table 4-1 given on page 4-20 of the AISC manual.
- Table 4-1 gives the stiffness reduction factor (τ) as a function of the yield stress F_y and the stress P_u/A_g in the column, where P_u is factored design load (analysis)

<u>EXAMPLE 3.7</u> Calculate the effective length factor for a W10 x 60 column AB made from 50 ksi steel in the unbraced frame shown below. Column AB has a design factor load P_u = 450 kips. The columns are oriented such that major axis bending occurs in the plane of the frame. The columns are braced *continuously along the length* for out-of-plane buckling. Assume that the same column section is used for the story above



<u>Solution</u>

Step I. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (*sidesway uninhibited*) frame.
- L_y = 0 ft.
- K_y has no meaning because out-of-plane buckling is not possible.
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.

Step II (a) - Calculate K_x

- I_{xx} of W 14 x 74 = 796 in^4 $\qquad$ I_{xx} of W 10 x 60 = 341 in^4

- $$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{341}{12\times12} + \frac{341}{15\times12}}{\frac{796}{18\times12} + \frac{796}{20\times12}} = \frac{4.2625}{7.002} = 0.609$$
- $G_B = 10$ - for pin support, see note on Page 16.1-191
- Using G_A and G_B: $K_x = 1.8$ - from Alignment Chart on Page 16.1-192
- Note, K_x is greater than 1.0 because it is an unbraced frame.

Step II (b) - Calculate $K_{x-inelastic}$ using stiffness reduction factor method

- Reduction in the flexural rigidity of the column due to residual stress effects
  - First calculate, $P_u / A_g = 450 / 17.6 = 25.57$ ksi
  - Then go to Table 4-1 on page 4-20 of the manual, and read the value of stiffness reduction factor for $F_y = 50$ ksi and $P_u/A_g = 25.57$ ksi.
  - Stiffness reduction factor = $\tau = 0.833$
- $G_{A\text{-inelastic}} = \tau \times G_A = 0.833 \times 0.609 = 0.507$
- $G_B = 10$ - for pin support, see note on Page **16.1**-191
- Using $G_{A\text{-inelastic}}$ and G_B, $K_{x\text{-inelastic}} = 1.75$ - alignment chart on Page **16.1**-192
- Note: *You can combine Steps II (a) and (b) to calculate the $K_{x\text{-inelastic}}$ directly.* You don't need to calculate elastic K_x first. It was done here for demonstration purposes.
- Note that $K_{x\text{-inelastic}} < K_x$. This is in agreement with the fact that the beams offer better resistance to the *inelastic* column AB because it has reduced flexural rigidity.

Step III – Design strength of the column

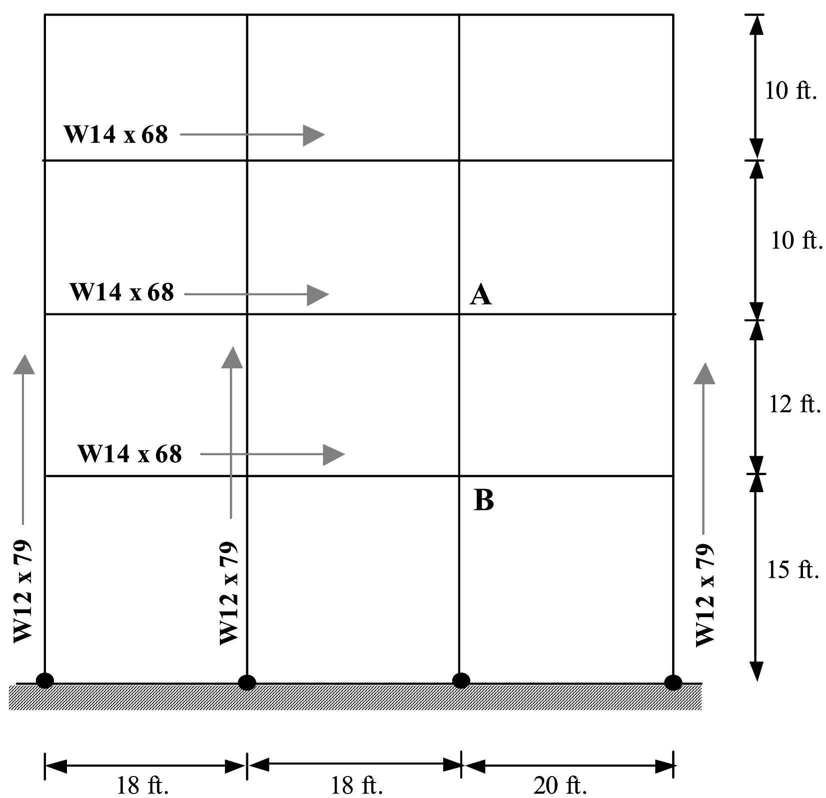
- $K_xL_x = 1.75 \times 15 = 26.25$ ft.
  - r_x / r_y for W10x60 = 1.71 - from Table **4**-2, see page **4**-26
  - $(KL)_{eq} = 26.25/1.71 = 15.35$ ft.

- $\phi_c P_n$ for X-axis buckling = 513.9 kips - from Table **4**-2, see page **4**-26
- Section slightly over-designed for P_u = 450 kips.

Column design strength = $\phi_c P_n$ = 513.9 kips

EXAMPLE 3.8:

- Design Column AB of the frame shown below for a design load of 500 kips.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced at each story level for out-of-plane buckling.
- Assume that the same column section is used for the stories above and below.



Step I - Determine the design load and assume the steel material.

- Design Load = P_u = 500 kips
- Steel yield stress = 50 ksi (A992 material)

Step II. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

- It is an unbraced (sidesway uninhibited) frame.

- $L_x = L_y = 12$ ft.
- $K_y = 1.0$
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.
- <u>Need to select a section to calculate K_x</u>

Step III - Select a column section

- *Assume minor axis buckling governs.*
- $K_y\ L_y = 12$ ft.
- See Column Tables in AISC-LRFD manual
  <u>Select section W12x53</u>
- $\phi_c P_n$ for y-axis buckling = 518 kips

Step IV - Calculate $K_{x\text{-inelastic}}$

- I_{xx} of W 12 x 53 =425 in^4 $\qquad$ I_{xx} of W14x68 = 753 in^4
- Account for the reduced flexural rigidity of the column due to residual stress effects
  - $P_u/A_g = 500 / 15.6 = 32.05$ ksi
  - Stiffness reduction factor = $\tau = 0.58$
- $$G_A = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.58 \times \left(\frac{425}{10 \times 12} + \frac{425}{12 \times 12} \right)}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{3.766}{6.360} = 0.592$$
- $$G_B = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.58 \times \left(\frac{425}{12 \times 12} + \frac{425}{15 \times 12} \right)}{\frac{723}{18 \times 12} + \frac{723}{20 \times 12}} = \frac{3.0812}{6.360} = 0.484$$
- Using G_A and G_B: $K_{x\text{-inelastic}} = 1.2$ $\qquad$ - from Alignment Chart

Step V - Check the selected section for X-axis buckling

- $K_x\ L_x = 1.2 \times 12 = 14.4$ ft.
- r_x / r_y for W12x53 = 2.11

- Calculate $(KL)_{eq}$ to determine strength ($\phi_c P_n$) for X-axis buckling

  $(KL)_{eq} = 14.4 / 2.11 = 6.825$ ft.
- From the column design tables, $\phi_c P_n$ for X-axis buckling = 612.3 kips

Step VI. Check the local buckling limits

For the flanges, $b_f/2t_f = 8.69 \quad < \quad \lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

For the web, $h/t_w = 28.1 \quad < \quad \lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

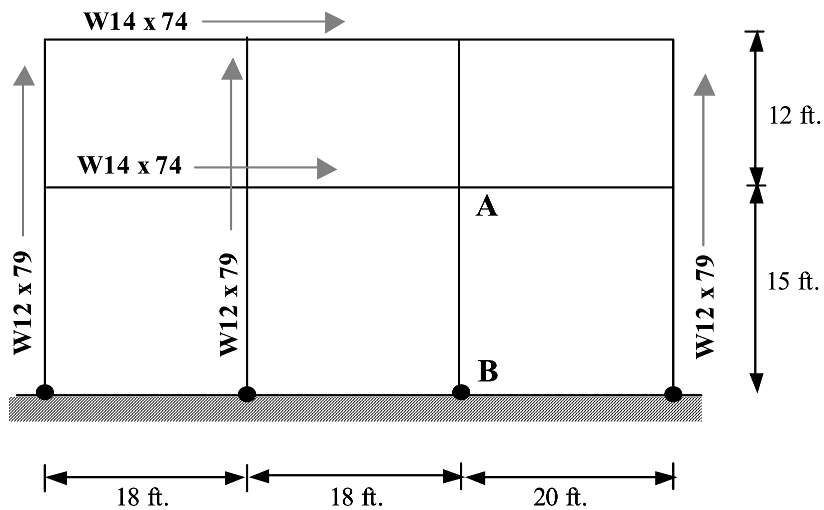
Therefore, the section is non-compact. OK, local buckling is not a problem

Step VII - Summarize the solution

$L_x = L_y = 12$ ft.	*$K_y = 1.0$*
$K_x = 1.2$ (inelastic buckling - sway frame-alignment chart method)	
$\phi_c P_n$ for Y-axis buckling = 518 kips	
$\phi_c P_n$ for X-axis buckling = 612.3 kips	
Y-axis buckling governs the design.	
Selected Section is W12 x 53 made from 50 ksi steel.	

EXAMPLE 3.9

- Design Column AB of the frame shown below for a design load of 450 kips.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced continuously along the length for out-of-plane buckling.
- Assume that the same column section is used for the story above.



Step I - Determine the design load and assume the steel material.

- Design Load = P_u = 450 kips
- Steel yield stress = 50 ksi

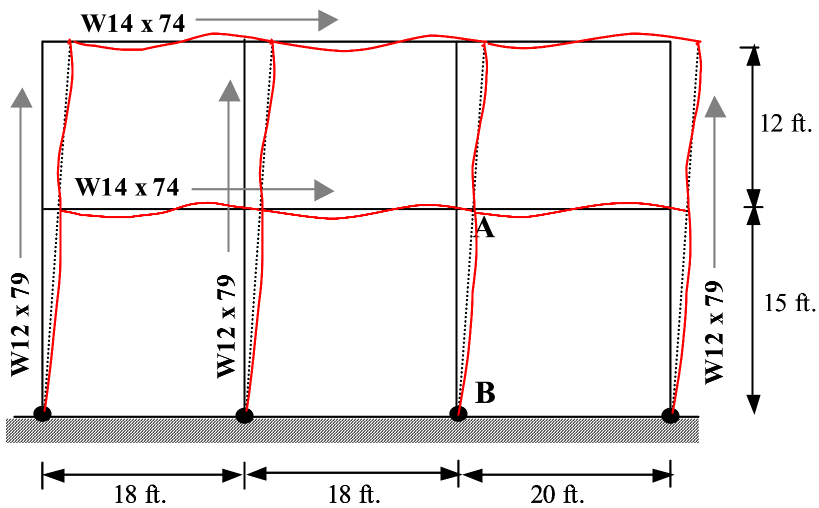
Step II. Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

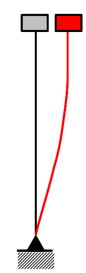
- It is an unbraced (*sidesway uninhibited*) frame.
- L_y = 0 ft.
- K_y has no meaning because out-of-plane buckling is not possible.
- K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.
- Need to calculate K_x using alignment charts.
- Need to select a section to calculate K_x

Step III. Select a section

- There is no help from the minor axis to select a section
- Need to assume K_x to select a section.

  See Figure below:





- The best case scenario for K_x is when the beams connected at joint A have infinite flexural stiffness (rigid). In that case $K_x = 2.0$ from Table C-C2.1
- Actually, the beams don't have infinite flexural stiffness. Therefore, calculated K_x should be greater than 2.0.
- To select a section, assume $K_x = 2.0$
  - $K_xL_x = 2.0 \times 15.0$ ft. = 30.0 ft.
- Need to be able to calculate $(KL)_{eq}$ to be able to use the column design tables to select a section. Therefore, need to assume a value of r_x/r_y to select a section.
  - See the W10 column tables on page 4-26.
  - Assume $r_x/r_y = 1.71$, which is valid for W10 x 49 to W10 x 68.
- $(KL)_{eq} = 30.0/1.71 = 17.54$ ft.
  - Obviously from the Tables, for $(KL)_{eq} = 17.5$ ft., W10 x 60 is the first section that will have $\phi_c P_n > 450$ kips
- Select W10x60 with $\phi_c P_n = 457.7$ kips for $(KL)_{eq} = 17.5$ ft.

Step IV - Calculate $K_{x\text{-inelastic}}$ using selected section

- I_{xx} of W 14 x 74 = 796 in^4 I_{xx} of W 10 x 60 = 341 in^4
- Account for the reduced flexural rigidity of the column due to residual stress effects
  - P_u/A_g = 450 / 17.6 = 25.57 ksi
  - Stiffness reduction factor = τ = 0.833
- $$G_A = \frac{\tau \times \sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{0.833 \times \left(\frac{341}{12 \times 12} + \frac{341}{15 \times 12}\right)}{\frac{796}{18 \times 12} + \frac{796}{20 \times 12}} = \frac{3.550}{7.002} = 0.507$$
- $G_B = 10$ - for pin support
- Using G_A and G_B: $K_{x\text{-inelastic}}$ = 1.75 - from Alignment Chart on Page 3-6
- Calculate value of $K_{x\text{-inelastic}}$ is less than 2.0 (the assumed value) because G_B was assumed to be equal to 10 instead of ∞

Step V - Check the selected section for X-axis buckling

- $K_x L_x$ = 1.75 x 15 = 26.25 ft.
  - r_x / r_y for W10x60 = 1.71
  - $(KL)_{eq}$ = 26.25/1.71 = 15.35 ft.
  - $(\phi_c P_n)$ for X-axis buckling = 513.9 kips
- Section slightly over-designed for P_u = 450 kips.
- *W10 x 54 will probably be adequate, Student should check by calculating K_x inelastic and $\phi_c P_n$ for that section.*

Step VI. Check the local buckling limits

For the flanges, $b_f/2t_f$ = 7.41 < $\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

For the web, h/t_w = 18.7 < $\lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

Therefore, the section is non-compact. OK, local buckling is not a problem

- **Step VII - Summarize the solution**

L_y = 0 ft.	*K_y = no buckling*
K_x = 1.75 (inelastic buckling - sway frame - alignment chart method)	
$\phi_c P_n$ for X-axis buckling = 513.9 kips	
X-axis buckling governs the design.	
Selected section is W10 x 60 *(W10 x 54 will probably be adequate).*	

3.9 DESIGN OF SINGLY SYMMETRIC CROSS-SECTIONS

- So far, we have been talking about doubly symmetric wide-flange (I-shaped) sections and channel sections. These rolled shapes always fail by *flexural* buckling.
- Singly symmetric (Tees and double angle) sections fail either by *flexural* buckling about the axis of non-symmetry or by *flexural-torsional* buckling about the axis of symmetry and the longitudinal axis.

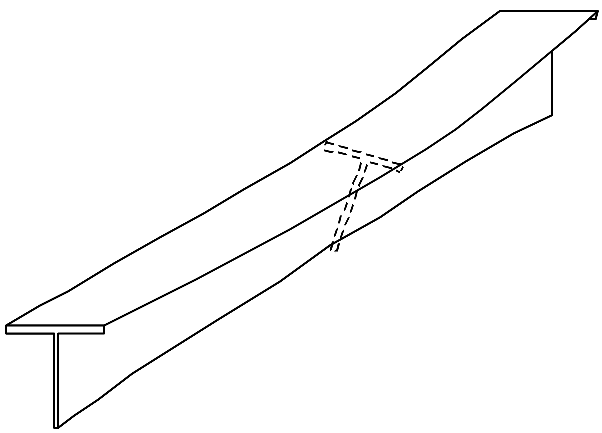


Figure 6(a). Flexural buckling

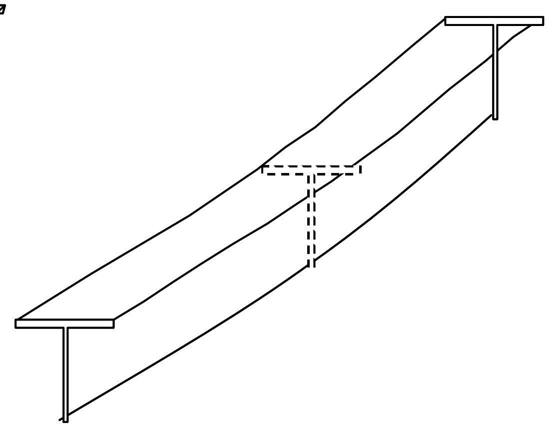
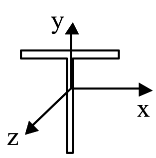


Figure 6(b). Flexural-torsional buckling



Flexural buckling will occur about the x-axis

Flexural-torsional buckling will occur about the y and z-axis

Smaller of the two will govern the design strength

Figure 6(c). Singly symmetric cross-section

- The AISC specification for flexural-torsional buckling is given by Spec. E3.

$$\text{Design strength} = \phi_c P_n = 0.85\ A_g\ F_{crft} \tag{1}$$

$$\text{Where, } F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2\ H}\right)\left[1 - \sqrt{1 - \frac{4\ F_{cry}\ F_{crz}\ H}{(F_{cry} + F_{crz})^2}}\right] \tag{2}$$

F_{cry} = critical stress for buckling about the y-axis, see Spec. E2. (3)

$$F_{crz} = \frac{GJ}{A\,\bar{r}_o^{\,2}} \tag{4}$$

$$\bar{r}_o^2 = \text{polar radius of gyration about shear center (in.)} = y_o^2 + \frac{I_x + I_y}{A} \tag{5}$$

$$H = 1 - \frac{y_o^2}{\bar{r}_o^2} \tag{6}$$

y_o = distance between shear center and centroid (in.) (7)

- The section properties for calculating the flexural-torsional buckling strength F_{crft} are given as follows:

  - $G = \dfrac{E}{2(1+\upsilon)}$
  - J, $\bar{r}_o^2$, H are given for WT shapes in Table 1-32 on page 1-101 to page 1-105
  - $\bar{r}_o^2$, H are given for double-angle shapes in Table 1-35 on page 1-108 to 1-110
  - J for single-angle shape in Table 1-31 on page 1-98 to 1-100. ($J_{2L} = 2 \times J_L$)

- The design tables for WT shapes given in Table 4-5 on page 4-35 to 4-47. These design tables include the axial compressive strength for flexural buckling about the x axis and flexural-torsional buckling about the y and z axis.

<u>EXAMPLE 3.10</u> Calculate the design compressive strength of a WT10.5 x 66. The effective length with respect to x-axis is 25ft. 6in. The effective length with respect to the y-axis is 20 ft. and the effective length with respect to z-axis is 20ft. A992 steel is used.

<u>Solution</u>

- **Step I.** Buckling strength about x-axis

$$\lambda_{c\text{-}x} = \frac{K_x\ L_x}{r_x\ \pi}\sqrt{\frac{F_y}{E}} = \frac{306}{3.06 \times 3.1416}\sqrt{\frac{50}{29000}} = 1.321$$

$\phi_c P_n = 0.85 \text{ x } (0.658)^{1.321^2} \text{ x } 50 \text{ x } 19.4 = 397.2$ kips

Values for A_g and r_x from page 4-41 of the manual. Compare with tabulated design strength for buckling about x-axis in Table 4-5

- **Step II.** Flexural-torsional buckling about the y and z axes

- Calculate F_{cry} and F_{crz} then calculate F_{crft} and $\phi_c P_n$

- $$\lambda_{c\text{-}y} = \frac{K_y\ Ly}{r_y\ \pi}\sqrt{\frac{F_y}{E}} = \frac{240}{2.93 \times 3.1416}\sqrt{\frac{50}{29000}} = 1.083$$

- $F_{cry} = (0.658)^{1.083^2} \text{ x } 50 = 30.6$ ksi

- $F_{crz} = GJ/A\bar{r}_o^2 = 11{,}153 \text{ x } 5.62/(4.60^2 \text{ x } 19.4) = 152.69$

- $$F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2\ H}\right)\left[1 - \sqrt{1 - \frac{4\ F_{cry}\ F_{crz}\ H}{(F_{cry} + F_{crz})^2}}\right] = \left(\frac{30.6 + 152.7}{2 \times 0.844}\right)\left[1 - \sqrt{1 - \frac{4 \times 30.6 \times 152.7 \times 0.844}{(30.6 + 152.7)^2}}\right]$$

  $F_{crft} = 108.58 \text{ x } 0.272 = 29.534$ ksi

- $\phi_c P_n = 0.85 \text{ x } F_{crft} \text{ x } A_g = 0.85 \text{ x } 29.534 \text{ x } 19.4 = 487$ kips

Values for J, $\bar{r}_o^2$, and H were obtained from flexural-torsional properties given in Table 1-32 on page 1-102. Compare the $\phi_c P_n$ value with the value reported in Table 4-5 (page 4-41) of the AISC manual.

- **Step III.** Design strength and check local buckling

  Flanges: $b_f/2t_f$ = 12.4/(2 x 1.03) = 6.02 , which is < $\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

  Stem of Tee: d/t_w = 10.9/0.65 = 16.77, which is < $\lambda_r = 0.75 \times \sqrt{\frac{E}{F_y}} = 18.08$

  Local buckling is not a problem. Design strength = 397.2 kips. X-axis flexural buckling governs.

3.10 DESIGN OF DOUBLE ANGLE SECTIONS

- Double-angle sections are very popular as compression members in trusses and bracing members in frames.
  - These sections consist of two angles placed back-to-back and connected together using bolts or welds.
  - You have to make sure that the two single angle sections are connected such that they do not buckle (individually) between the connections along the length.
  - The AISC specification E4.2 requires that $\mathbf{Ka/r_z}$ of the individual single angles < ¾ of the *governing* **KL/r** of the double angle.
    - where, a is the distance between connections and r_z is the smallest radius of gyration of the single angle (see dimensions in Table 1-7)
- Double-angle sections can fail by flexural buckling about the x-axis or flexural torsional buckling about the y and z axes.

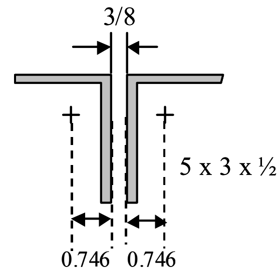
- For flexural buckling about the x-axis, the moment of inertia $I_{x\text{-}2L}$ of the double angle will be equal to two times the moment of inertia $I_{x\text{-}L}$ of each single angle.
- For flexural torsional buckling, there is a slight problem. The double angle section will have some *additional flexibility* due to the intermittent connectors. This added flexibility will depend on the connection parameters.

- According to AISC Specification E4.1, a modified $(KL/r)_m$ must be calculated for the double angle section for buckling about the y-axis to account for this added flexibility
  - Intermediate connectors that are snug-tight bolted $\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{a}{r_z}\right)^2}$
  - Intermediate connectors that are welded or fully tensioned bolted:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_y}\right)^2}$$

where, α = separation ratio = $h/2r_y$

h = distance between component centroids in the y direction



EXAMPLE 3.11 Calculate the design strength of the compression member shown in the figure. Two angles, 5 x 3 x ½ are oriented with the long legs back-to-back and separated by 3/8 in. The effective length KL is 16 ft. A36 steel is used. Assume three welded intermediate connectors

Solution

Step I. Determine the relevant properties from the AISC manual

Property	Single angle	Double angle
A_g	3.75 in^2	7.5 in^2
r_x	1.58 in.	1.58 in.
r_y	0.824 in.	1.24 in.
r_z	0.642 in.	-----
J	0.322 in^4	0.644 in^4
$\bar{r}_o^2$		2.51 in.
H		0.646
AISC Page no.	**1**-36, **1**-37, **1**-99	**1**-75, **1**-109

Step II. Calculate the x-axis buckling strength

- KL/r_x = 16 x 12 /1.58 = 120.8
- $\lambda_{c\text{-}x} = \frac{K_x\ L_x}{r_x\ \pi}\sqrt{\frac{F_y}{E}} = \frac{120.8}{3.1416}\sqrt{\frac{36}{29000}} = 1.355$
- $\phi_c P_n = 0.85 \times (0.658)^{1.355^2} \times 36 \times (2 \times 3.75) = 106$ kips

Step III. Calculate $(KL/r)_m$ for y-axis buckling

- (KL/r) = 16 x 12/1.24 = 154.8

- $a/r_z = 48/0.648 = 74.07$

  $a/r_z = 74.07 < 0.75 \text{ x } KL/r = 0.75 \text{ x } 154.8 = 115.2$ (OK!)

- $\alpha = h/2r_y = (2 \text{ x } 0.75 + 0.375)/(2 \text{ x } 0.829) = 1.131$

- $$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_y}\right)^2}$$

  $$= \sqrt{(154.8)_o^2 + 0.82\frac{1.131^2}{1+1.131^2}\left(\frac{48}{0.829}\right)^2} = 158.5$$

Step IV. Calculate flexural torsional buckling strength.

- $$\lambda_{c\text{-}y} = \left(\frac{KL}{r}\right)_m \times \frac{1}{\pi} \times \sqrt{\frac{F_y}{E}} = 1.778$$

- $$F_{cry} = \frac{0.877}{\lambda_{c-y}^2} \times F_y = \frac{0.877}{1.778^2} \times 36 = 9.987 \text{ ksi}$$

- $$F_{crz} = \frac{GJ}{A\bar{r}_o^2} = \frac{11{,}200 \times 0.644}{7.5 \times 2.51^2} = 151.4 \text{ ksi}$$

- $$F_{crft} = \left(\frac{F_{cry} + F_{crz}}{2\,H}\right)\left[1 - \sqrt{1 - \frac{4\,F_{cry}\,F_{crz}\,H}{(F_{cry} + F_{crz})^2}}\right] = \left(\frac{9.987 + 151.4}{2 \times 0.646}\right)\left[1 - \sqrt{1 - \frac{4 \times 9.987 \times 151.4 \times 0.646}{(9.987 + 151.4)^2}}\right]$$

  $F_{crft} = 9.748$ ksi

- $\phi_c P_n = 0.85 \text{ x } F_{crft} \text{ x } A_g = 0.85 \text{ x } 9.748 \text{ x } 7.50 = 62.1$ kips

Flexural torsional buckling strength controls. The design strength of the double angle member is 62.1 kips.

Step V. Compare with design strengths in Table 4-10 (page **4-**84) of the AISC manual

- $\phi_c P_n$ for x-axis buckling with unsupported length = 16 ft. = 106 kips

- $\phi_c P_n$ for y-z axis buckling with unsupported length = 16 ft. = 61.3 kips

These results make indicate excellent correlation between the calculations in steps II to IV and the tabulated values.

Design tables for double angle compression members are given in the AISC manual. See Tables 4-9, 4-10, and 4-11 on pages 4-78 to **4**-93

- In these Tables F_y = 36 ksi
- Back to back distance = 3/8 in.
- Design strength for buckling about x axis
- Design strength for flexural torsional buckling accounting for the *modified* slenderness ratio depending on the number of intermediate connectors.
- These design Tables can be used to design compression members as double angle sections.

Chapter 4. TENSION MEMBER DESIGN

4.1 INTRODUCTORY CONCEPTS

- Stress: The stress in an axially loaded tension member is given by Equation (4.1)

$$f = \frac{P}{A} \tag{4.1}$$

where, P is the magnitude of load, and

A is the cross-sectional area normal to the load

- The stress in a tension member is uniform throughout the cross-section except:
  - near the point of application of load, and
  - at the cross-section with holes for bolts or other discontinuities, etc.
- For example, consider an 8 x ½ in. bar connected to a gusset plate and loaded in tension as shown below in Figure 4.1

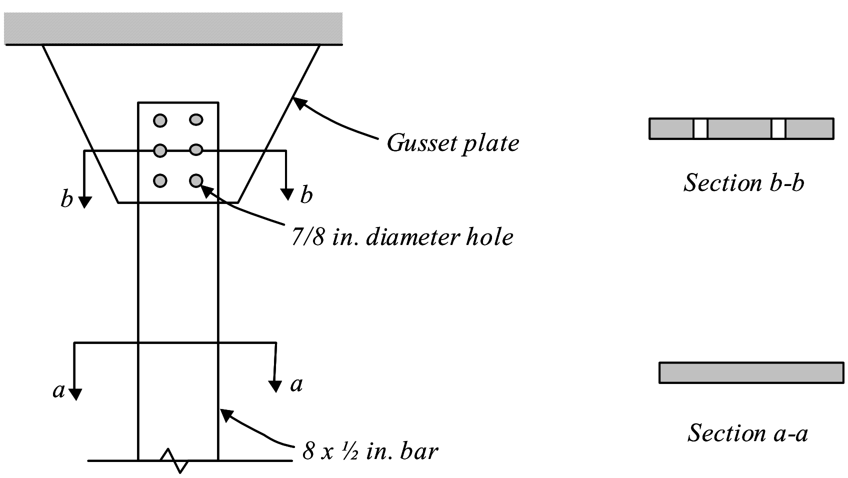


Figure 4.1 Example of tension member.

- Area of bar at section $a - a$ = 8 x ½ = 4 in^2
- Area of bar at section $b - b$ = (8 – 2 x 7/8) x ½ = 3.12 in^2

- Therefore, by definition (Equation 4.1) the reduced area of section *b – b* will be subjected to higher stresses
- However, the reduced area and therefore the higher stresses will be localized around section b – b.
- The unreduced area of the member is called its gross area = A_g
- The reduced area of the member is called its net area = A_n

4.2 STEEL STRESS-STRAIN BEHAVIOR

- The stress-strain behavior of steel is shown below in Figure 4.2

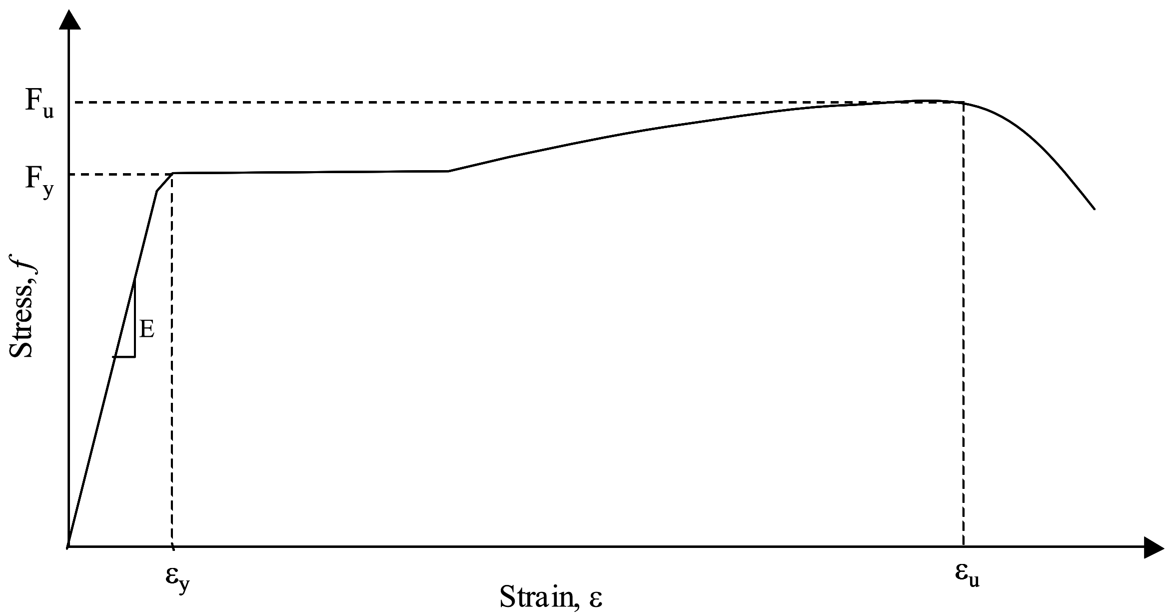


Figure 4.2 Stress-strain behavior of steel

- In Figure 4.2, E is the elastic modulus = 29000 ksi.

  F_y is the yield stress and F_u is the ultimate stress

  ε_y is the yield strain and ε_u is the ultimate strain

- Deformations are caused by the strain ε. Figure 4.2 indicates that the structural deflections will be small as long as the material is elastic ($f < F_y$)
- Deformations due to the strain ε will be large after the steel reaches its yield stress F_y.

4.3 DESIGN STRENGTH

- A tension member can fail by reaching one of two limit states:

  (1) excessive deformation; or (2) fracture
- Excessive deformation can occur due to the yielding of the gross section (for example section a-a from Figure 4.1) along the length of the member
- Fracture of the net section can occur if the stress at the net section (for example section b-b in Figure 4.1) reaches the ultimate stress F_u.
- The objective of design is to prevent these failure before reaching the ultimate loads on the structure (*Obvious*).
- This is also the load and resistance factor design approach recommended by AISC for designing steel structures

4.3.1 Load and Resistance Factor Design

The load and resistance factor design approach is recommended by AISC for designing steel structures. It can be understood as follows:

<u>Step I. Determine the ultimate loads acting on the structure</u>

- The values of D, L, W, etc. given by ASCE 7-98 are nominal loads (not maximum or ultimate)
- During its design life, a structure can be subjected to some maximum or ultimate loads caused by combinations of D, L, or W loading.

- The ultimate load on the structure can be calculated using factored load combinations, which are given by ASCE and AISC (see pages 2-10 and 2-11 of AISC manual). The most relevant of these load combinations are given below:

$$1.4\ D \tag{4.2 – 1}$$

$$1.2\ D + 1.6\ L + 0.5\ (L_r \text{ or } S) \tag{4.2 – 2}$$

$$1.2\ D + 1.6\ (L_r \text{ or } S) + (0.5\ L \text{ or } 0.8\ W) \tag{4.2 – 3}$$

$$1.2\ D + 1.6\ W + 0.5\ L + 0.5\ (L_r \text{ or } S) \tag{4.2 – 4}$$

$$0.9\ D + 1.6\ W \tag{4.2 – 5}$$

Step II. Conduct linear elastic structural analysis

- Determine the design forces (P_u, V_u, and M_u) for each structural member

Step III. Design the members

- The failure (design) strength of the designed member must be greater than the corresponding design forces calculated in Step II. See Equation (4.3) below:

$$\phi R_n > \sum \gamma_i\ Q_i \tag{4.3}$$

- Where, R_n is the calculated failure strength of the member
- ϕ is the resistance factor used to account for the reliability of the material behavior and equations for R_n
- Q_i is the nominal load
- γ_i is the load factor used to account for the variability in loading and to estimate the ultimate loading condition.

4.3.2 Design Strength of Tension Members

- Yielding of the gross section will occur when the stress f reaches F_y.

$$f = \frac{P}{A_g} = F_y$$

Therefore, nominal yield strength = $P_n = A_g F_y$ (4.4)

Factored yield strength = $\phi_t P_n$ (4.5)

where, $\phi_t = 0.9$ for tension yielding limit state

- See the AISC manual, section on specifications, Chapter D (page 16.1 –24)
- Facture of the net section will occur after the stress on the net section area reaches the ultimate stress F_u

$$f = \frac{P}{A_e} = F_u$$

Therefore, nominal fracture strength = $P_n = A_e F_u$

Where, A_e is the effective net area, which may be equal to the net area or smaller.

The topic of A_e will be addressed later.

Factored fracture strength = $\phi_t A_e F_u$ (4.6)

Where, $\phi_t = 0.75$ for tension fracture limit state (See page 16.1-24 of AISC manual)

4.3.3 Important notes

- Note 1. Why is fracture (& not yielding) the relevant limit state at the net section?

  Yielding will occur first in the net section. However, the deformations induced by yielding will be localized around the net section. These localized deformations will *not* cause excessive deformations in the complete tension member. Hence, yielding at the net section will *not* be a failure limit state.

- Note 2. Why is the resistance factor (ϕ_t) smaller for fracture than for yielding?

  The smaller resistance factor for fracture ($\phi_t = 0.75$ as compared to $\phi_t = 0.90$ for yielding) reflects the more serious nature and consequences of reaching the fracture limit state.

- Note 3. What is the design strength of the tension member?

The design strength of the tension member will be the lesser value of the strength for the two limit states (gross section yielding and net section fracture).

- Note 4. Where are the F_y and F_u values for different steel materials?

  The yield and ultimate stress values for different steel materials are noted in Table 2 in the *AISC* manual on pages *16.1–141* and *16.1–142*.

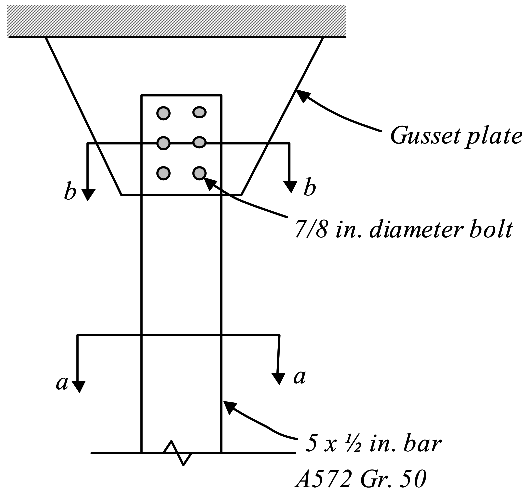
- Note 5. What are the most common steels for structural members?

  See Table 2-1 in the *AISC* manual on pages *2–24* and *2-25*. According to this Table: the preferred material for *W* shapes is *A992* (F_y = 50 ksi; F_u = 65 ksi); the preferred material for *C*, *L* , *M* and *S* shapes is *A36* (F_y = 36 ksi; F_u = 58 ksi). All these shapes are also available in *A572 Gr. 50* (F_y = 50 ksi; F_u = 65 ksi).

- Note 6. What is the amount of area to be deducted from the gross area to account for the presence of bolt-holes?

  - The *nominal* diameter of the hole (d_h) is equal to the bolt diameter (d_b) + 1/16 in.
  - However, the bolt-hole fabrication process damages additional material around the hole diameter.
  - Assume that the material damage extends 1/16 *in*. around the hole diameter.
  - Therefore, for calculating the net section area, assume that the gross area is *reduced by a hole diameter* equal to the nominal hole-diameter + 1/16 in.

Example 3.1 A 5 x ½ bar of A572 Gr. 50 steel is used as a tension member. It is connected to a gusset plate with six 7/8 in. diameter bolts as shown in below. Assume that the effective net area A_e equals the actual net area A_n and compute the tensile design strength of the member.



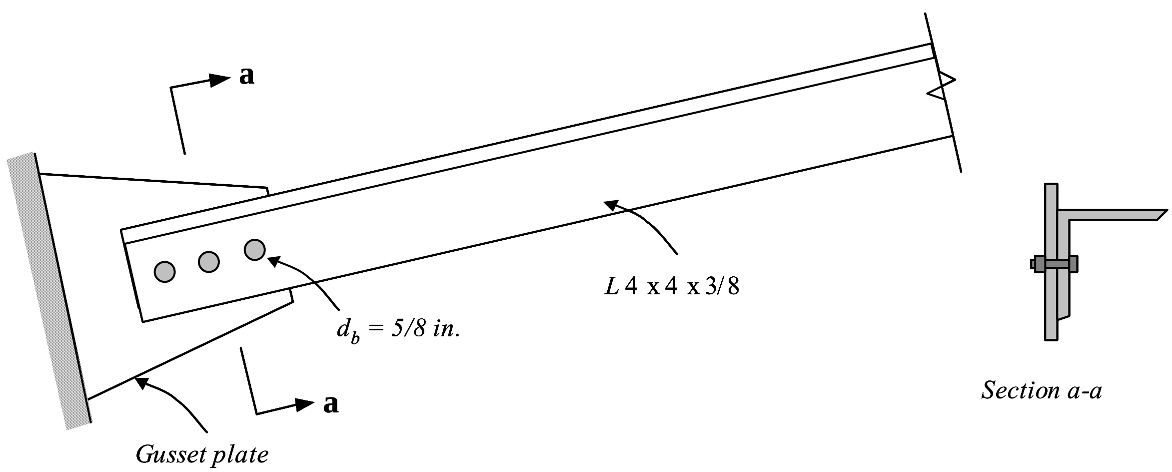
Solution

- Gross section area = A_g = 5 x ½ = 2.5 in^2
- Net section area (A_n)
  - Bolt diameter = d_b = 7/8 in.
  - Nominal hole diameter = d_h = 7/8 + 1/16 in. = 15/16 in.
  - Hole diameter for calculating net area = 15/16 + 1/16 in. = 1 in.
  - Net section area = A_n = (5 – 2 x (1)) x ½ = 1.5 in^2
- Gross yielding design strength = $\phi_t P_n = \phi_t F_y A_g$
  - Gross yielding design strength = 0.9 x 50 ksi x 2.5 in^2 = 112.5 kips
- Fracture design strength = $\phi_t P_n = \phi_t F_u A_e$
  - Assume $A_e = A_n$ (only for this problem)
  - Fracture design strength = 0.75 x 65 ksi x 1.5 in^2 = 73.125 kips
- Design strength of the member in tension = smaller of 73.125 kips and 112.5 kips

- Therefore, design strength = 73.125 kips (*net section fracture controls*).

Example 3.2 A single angle tension member, *L* 4 x 4 x 3/8 in. made from A36 steel is connected to a gusset plate with 5/8 in. diameter bolts, as shown in Figure below. The service loads are 35 kips dead load and 15 kips live load. Determine the adequacy of this member using AISC specification. Assume that the effective net area is 85% of the computed net area. (*Calculating the effective net area will be taught in the next section*).

- Gross area of angle = A_g = 2.86 in^2 (*from Table 1-7 on page 1-36 of AISC*)



- Net section area = A_n
  - Bolt diameter = 5/8 in.
  - Nominal hole diameter = 5/8 + 1/16 = 11/16 in.
  - Hole diameter for calculating net area = 11/16 + 1/16 = 3/4 in.
  - Net section area = A_g – (3/4) x 3/8 = 2.86 – 3/4 x 3/8 = 2.579 in^2
- Effective net area = A_e = 0.85 x 2.579 in^2 = 2.192 in^2
- Gross yielding design strength = ϕ_t A_g F_y = 0.9 x 2.86 in^2 x 36 ksi = 92.664 kips
- Net section fracture = ϕ_t A_e F_u = 0.75 x 2.192 in^2 x 58 ksi = 95.352 kips

- Design strength = 92.664 kips (gross yielding governs)
- Ultimate (design) load acting for the tension member = P_u
  - The ultimate (design) load can be calculated using factored load combinations given on page *2-11* of the *AISC* manual, or Equations (*4.2-1 to 4.2-5*) of notes (see pg. 4)
  - According to these equations, two loading combinations are important for this problem. These are: (1) 1.4 *D*; and (2) 1.2 *D* + 1.6 *L*
  - The corresponding ultimate (design) loads are:

    1.4 x (P_D) = 1.4 (35) = 49 kips

    1.2 (P_D) + 1.6 (P_L) = 66 kips (controls)
  - The ultimate design load for the member is 66 kips, where the *factored* dead + live loading condition controls.
- Compare the design strength with the ultimate design load
  - The design strength of the member (92.664 kips) is greater than the ultimate design load (66 kips).
  - ϕ_t P_n (92.664 kips) > P_u (66 kips)
- The *L* 4 x 4 x 3/8 in. made from A36 steel is adequate for carrying the factored loads.

4.4 EFFECTIVE NET AREA

- The <u>*connection*</u> has a significant influence on the performance of a tension member. A connection almost always weakens the member, and a measure of its influence is called joint efficiency.
- Joint efficiency is a function of: (a) material ductility; (b) fastener spacing; (c) stress concentration at holes; (d) fabrication procedure; and (e) **shear lag**.

- All factors contribute to reducing the effectiveness but shear lag is the most important.
- Shear lag occurs when the tension force is not transferred *simultaneously* to all elements of the cross-section. This will occur when some elements of the cross-section are not connected.
- For example, see Figure 4.3 below, where only one leg of an angle is bolted to the gusset plate.

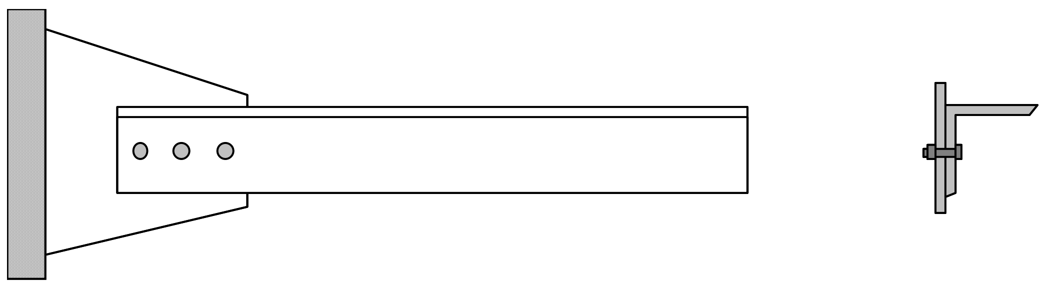


Figure 4.3 Single angle with bolted connection to only one leg.

- A consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed.
- Lengthening the connection region will reduce this effect
- Research indicates that shear lag can be accounted for by using a reduced or effective net area A_e
- Shear lag affects both bolted and welded connections. Therefore, the effective net area concept applied to both types of connections.
  - For bolted connection, the effective net area is $A_e = U\,A_n$
  - For welded connection, the effective net area is $A_e = U\,A_g$
- Where, the reduction factor U is given by:

$$U = 1 - \frac{\bar{x}}{L} \leq 0.9 \qquad (4.7)$$

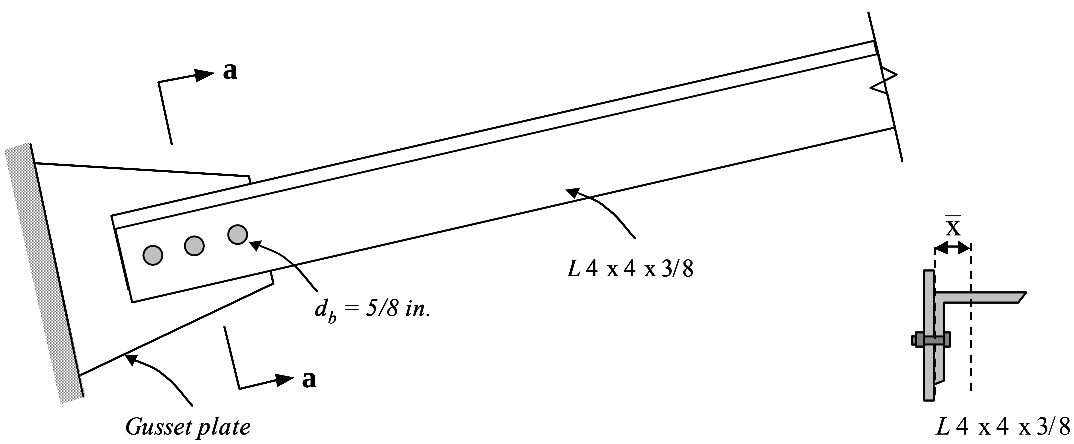
- Where, $\bar{x}$ is the distance from the centroid of the connected area to the plane of the connection, and L is the length of the connection.
  - If the member has two symmetrically located planes of connection, $\bar{x}$ is measured from the centroid of the nearest one – half of the area.
  - Additional approaches for calculating $\bar{x}$ for different connection types are shown in the *AISC* manual on page ***16.1-178.***
- The distance L is defined as the length of the connection in the direction of load.
  - For bolted connections, L is measured from the center of the bolt at one end to the center of the bolt at the other end.
  - For welded connections, it is measured from one end of the connection to other.
  - If there are weld segments of different length in the direction of load, L is the length of the longest segment.
  - Example pictures for calculating L are given on page ***16.1-179*** of *AISC.*

• The AISC manual also gives values of *U* that can be used instead of calculating $\bar{x}/L$.

- They are based on average values of $\bar{x}/L$ for various bolted connections.
- For W, M, and S shapes with width-to-depth ratio of at least 2/3 and for Tee shapes cut from them, if the connection is through the flanges with at least three fasteners per line in the direction of applied load ………………………………………………...$U = 0.90$
- For all other shapes with at least three fasteners per line …………………... $U = 0.85$
- For all members with only two fasteners per line …………………………… $U = 0.75$
- For better idea, see Figure 3.8 on page 41 of the Segui text-book.
- These values are acceptable but not the best estimate of U
- If used in the exam or homeworks, full points for calculating U will not be given

<u>Example 3.3</u> Determine the effective net area and the corresponding design strength for the single angle tension member of *Example 3.2*. The tension member is an *L* 4 x 4 x 3/8 in. made from A36 steel. It is connected to a gusset plate with 5/8 in. diameter bolts, as shown in Figure below. The spacing between the bolts is 3 in. center-to-center.

- Compare your results with those obtained for *Example 3.2*.



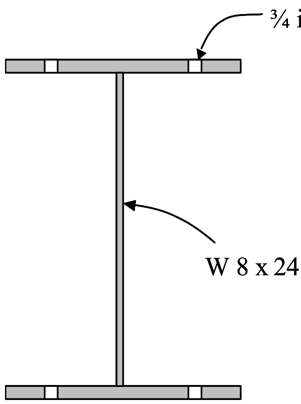
- Gross area of angle = A_g = 2.86 in^2 (*from Table 1-7 on page 1-36 of AISC*)
- Net section area = A_n
  - Bolt diameter = 5/8 in.
  - Hole diameter for calculating net area = 11/16 + 1/16 = 3/4 in.
  - Net section area = A_g – (3/4) x 3/8 = 2.86 – 3/4 x 3/8 = 2.579 in^2
- $\bar{x}$ is the distance from the centroid of the area connected to the plane of connection
  - For this case $\bar{x}$ is equal to the distance of centroid of the angle from the edge.
  - This value is given in the *Table 1-7 on page 1-36 of* the *AISC* manual.
  - $\bar{x}$ = 1.13 in.
- L is the length of the connection, which for this case will be equal to 2 x 3.0 in.

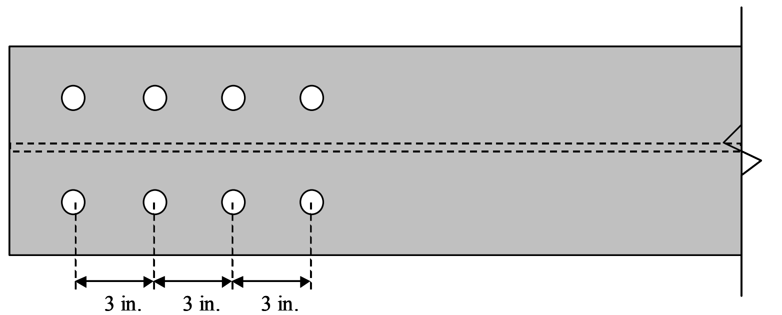
- L = 6.0 in.

- $U = 1 - \frac{\overline{x}}{L} = 1 - \frac{1.13}{6.0} = 0.8116$ in.
- Effective net area = A_e = 0.8116 x 2.579 in^2 = 2.093 in^2
- Gross yielding design strength = $\phi_t A_g F_y$ = 0.9 x 2.86 in^2 x 36 ksi = 92.664 kips
- Net section fracture = $\phi_t A_e F_u$ = 0.75 x 2.093 in^2 x 58 ksi = 91.045 kips
- Design strength = 91.045 kips (net section fracture governs)
- In Example 3.2
  - Factored load = P_u = 66.0 kips
  - Design strength = $\phi_t P_n$ = 92.66 kips (*gross section yielding governs*)
  - Net section fracture strength = $\phi_t P_n$ = 95.352 kips (*assuming* A_e = 0.85)
- Comparing Examples 3.2 and 3.3
  - Calculated value of *U* (0.8166) is less than the assumed value (0.85)
  - The assumed value was unconservative.
  - It is preferred that the *U* value be specifically calculated for the section.
  - After including the calculated value of *U*, net section fracture governs the design strength, but the member is still adequate from a design standpoint.

Example 3.4 Determine the design strength of an ASTM A992 W8 x 24 with four lines if ¾ in. diameter bolts in standard holes, two per flange, as shown in the Figure below.

Assume the holes are located at the member end and the connection length is 9.0 in. Also calculate at what length this tension member would cease to satisfy the slenderness limitation in LRFD specification B7





Holes in beam flange

Solution:

- For ASTM A992 material: F_y = 50 ksi; and F_u = 65 ksi
- For the W8 x 24 section:
  - A_g = 7.08 in^2 d = 7.93 in.
  - t_w = 0.285 in. b_f = 6.5 in.
  - t_f = 0.4 in. r_y = 1.61 in.
- Gross yielding design strength = $\phi_t P_n = \phi_t A_g F_y$ = 0.90 x 7.08 in^2 x 50 ksi = 319 kips
- Net section fracture strength = $\phi_t P_n = \phi_t A_e F_u$ = 0.75 x A_e x 65 ksi
  - $A_e = U A_n$ - for bolted connection
  - $A_n = A_g$ – (no. of holes) x (diameter of hole) x (thickness of flange)

    A_n = 7.08 – 4 x (diameter of bolt + 1/8 in.) x 0.4 in.

    A_n = 5.68 in^2

- $U = 1 - \frac{\bar{x}}{L} \leq 0.90$

- What is $\bar{x}$ for this situation?

  $\bar{x}$ *is the distance from the edge of the flange to the centroid of the half* (*T*) *section*

$$\bar{x} = \frac{(b_f \times t_f) \times \frac{t_f}{2} + (\frac{d - 2\,t_f}{2} \times t_w) \times (\frac{d + 2\,t_f}{4})}{b_f \times t_f + \frac{d}{2} \times t_w} = \frac{6.5 \times 0.4 \times 0.2 + 3.565 \times 0.285 \times 2.1825}{6.5 \times 0.4 + 3.565 \times 0.285} = 0.76$$

- $\bar{x}$ can be obtained from the dimension tables for Tee section *WT 4 x 12*. See page ***1-50*** and ***1-51*** of the AISC manual:

  $\bar{x}$ = 0.695 in.

- The calculated value is *not accurate* due to the deviations in the geometry

- $U = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.695}{9.0} = 0.923$

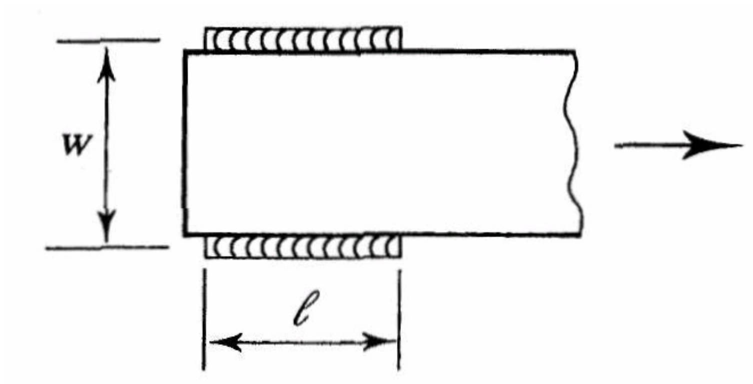
- But, $U \leq 0.90$. Therefore, assume $U = 0.90$

- Net section fracture strength = $\phi_t A_e F_u$ = 0.75 x 0.9 x 5.68 x 65 = 249.2 kips
- The design strength of the member is controlled by net section fracture = 249.2 kips
- According to LRFD specification B7, the maximum unsupported length of the member is limited to 300 r_y = 300 x 1.61 in. = 543 in. = 40.3 ft.

4.4.1 Special cases for welded connections

- If some elements of the cross-section are not connected, then A_e will be less than A_n
  - For a rectangular bar or plate A_e will be equal to A_n
  - However, if the connection is by longitudinal welds at the ends as shown in the figure below, then $A_e = UA_g$

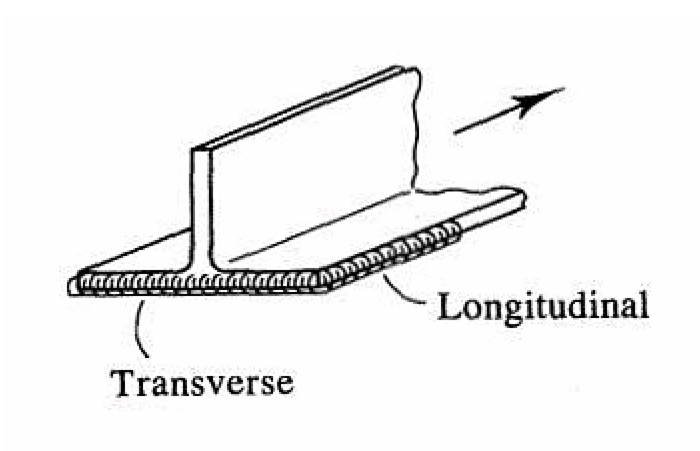
Where, $U = 1.0$ for $L \geq w$
$U = 0.87$ for $1.5\ w \leq L < 2\ w$
$U = 0.75$ for $w \leq L < 1.5\ w$
L = length of the pair of welds $\geq$ w
w = distance between the welds or width of plate/bar



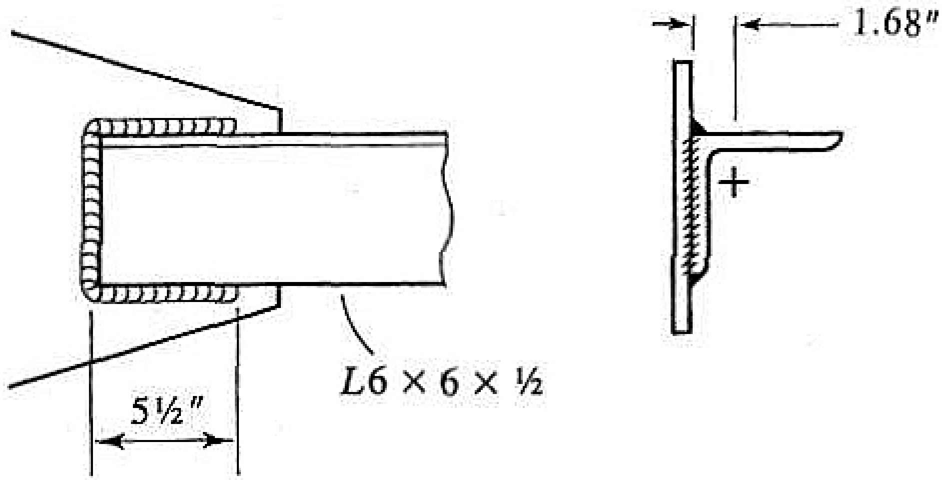
- AISC Specification B3 gives another special case for welded connections.

For any member connected by *transverse welds alone*,

A_e = area of the connected element of the cross-section

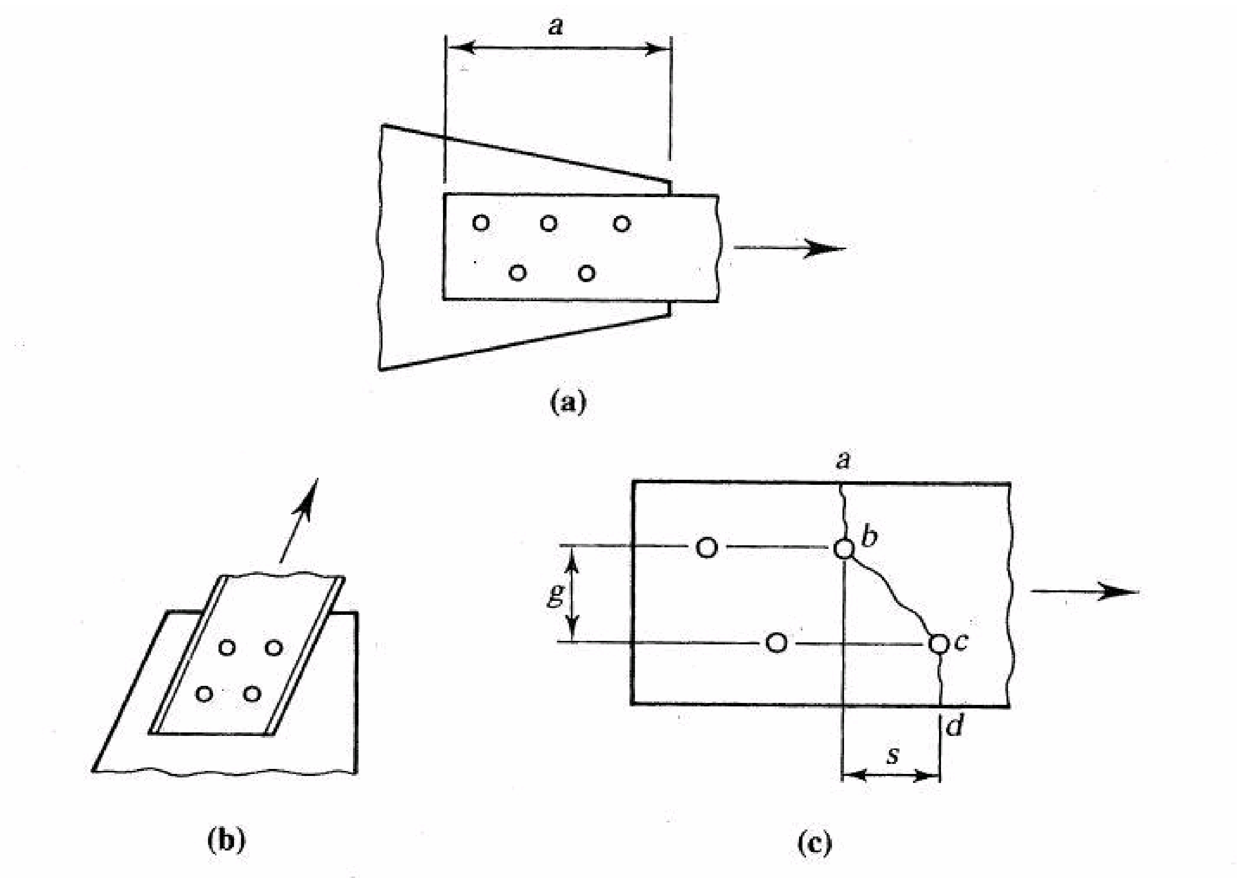


Example 3.5 Consider the welded single angle *L* 6x 6 x ½ tension member made from A36 steel shown below. Calculate the tension design strength.



Solution

- $A_g = 5.00 \text{ in}^2$
- $A_n = 5.00 \text{ in}^2$ - because it is a welded connection
- $A_e = U A_n$ - where, $U = 1 - \dfrac{\bar{x}}{L}$
  - $\bar{x} = 1.68$ in. for this welded connection
  - L = 6.0 in. for this welded connection
  - $U = 1 - \dfrac{1.168}{6.0} = 0.72$
- Gross yielding design strength = $\phi_t F_y A_g = 0.9 \times 36 \times 5.00 = 162$ kips
- Net section fracture strength = $\phi_t F_u A_e = 0.75 \times 58 \times 0.72 \times 5.00 = 156.6$ kips
- Design strength = 156.6 kips (*net section fracture governs*)



4.5 STAGGERED BOLTS

For a bolted tension member, the connecting bolts can be staggered for several reasons:

(1) To get more capacity by increasing the effective net area

(2) To achieve a smaller connection length

(3) To fit the geometry of the tension connection itself.

- For a tension member with staggered bolt holes (see example figure above), the relationship $f = P/A$ does not apply and the stresses are a combination of tensile and shearing stresses on the inclined portion *b-c*.

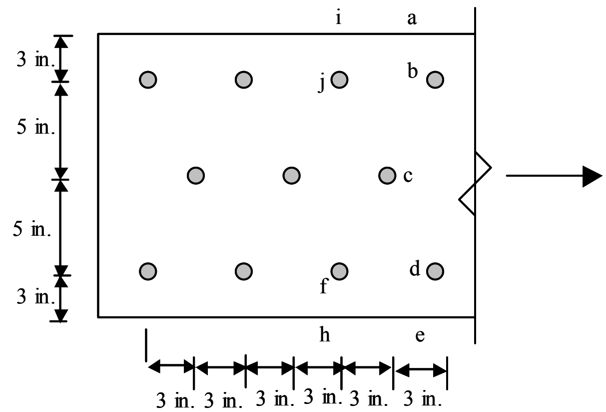
- Net section fracture can occur along any zig-zag or straight line. For example, fracture can occur along the inclined path *a-b-c-d* in the figure above. However, all possibilities must be examined.

- Empirical methods have been developed to calculate the net section fracture strength

  According to AISC Specification B2

  - net width = gross width $- \sum d + \sum \frac{s^2}{4g}$
  - where, d is the diameter of hole to be deducted (d_h + 1/16, or d_b + 1/8)
  - $s^2/4g$ is added for each gage space in the chain being considered
  - *s* is the longitudinal spacing (pitch) of the bolt holes in the direction of loading
  - *g* is the transverse spacing (gage) of the bolt holes perpendicular to loading dir.
  - net area (A_n) = net width x plate thickness
  - effective net area (A_e) = $U\,A_n$ where $U = 1 - \bar{x}/L$
  - net fracture design strength = $\phi_t\,A_e\,F_u$ ($\phi_t = 0.75$)

EXAMPLE 3.6 Compute the smallest net area for the plate shown below: The holes are for 1 in. diameter bolts.



- The effective hole diameter is 1 + 1/8 = 1.125 in.
- For line *a-b-d-e*

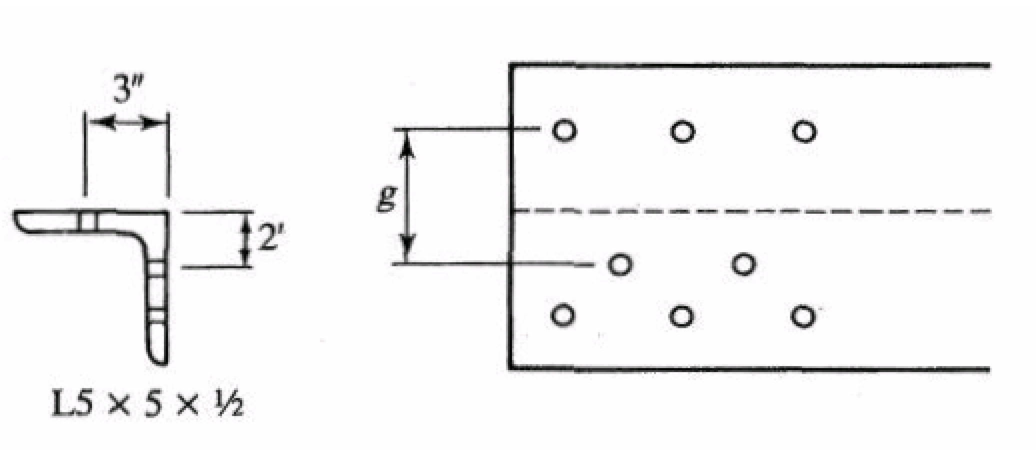
  $w_n = 16.0 – 2 (1.125) = 13.75$ in.
- For line *a-b-c-d-e*

  $w_n = 16.0 – 3 (1.125) + 2 \times 3^2 / (4 \times 5) = 13.52$ in.
- The line *a-b-c-d-e* governs:
- $A_n = t\ w_n = 0.75 (13.52) = 10.14$ in^2

Note

- Each fastener resists an equal share of the load
- Therefore different potential failure lines may be subjected to different loads.
- For example, line *a-b-c-d-e* must resist the full load, whereas *i-j-f-h* will be subjected to 8/11 of the applied load. The reason is that 3/11 of the load is transferred from the member before *i-j-f-h* received any load.

- **Staggered bolts in angles.** If staggered lines of bolts are present in both legs of an angle, then the net area is found by first unfolding the angle to obtain an equivalent plate. This plate is then analyzed like shown above.
  - The unfolding is done at the middle surface to obtain a plate with gross width equal to the sum of the leg lengths minus the angle thickness.
  - AISC Specification B2 says that any gage line crossing the heel of the angle should be reduced by an amount equal to the angle thickness.
  - See Figure below. For this situation, the distance g will be = 3 + 2 – ½ in.



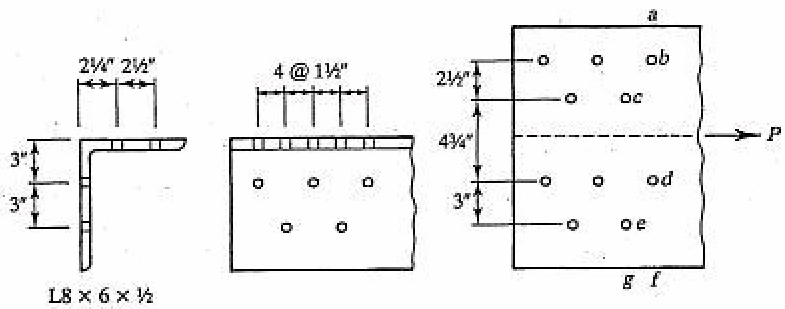
EXAMPLE 3.6

Find the design tensile strength of the angle shown in Figure 3.16. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

SOLUTION Compute the net width:

$$w_g = 8 + 6 - \tfrac{1}{2} = 13.5 \text{ in.}$$

FIGURE 3.16



Effective hole diameter = ⅞ + ⅛ = 1 in.

For line *abdf*,

$$w_n = 13.5 - 2(1) = 11.5 \text{ in.}$$

For line *abceg*,

$$w_n = 13.5 - 3(1) + \frac{(1.5)^2}{4(2.5)} = 10.73 \text{ in.}$$

Because $\frac{1}{10}$ of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only $\frac{9}{10}$ of the load. Therefore the net width of 10.73 inch should be multiplied by $\frac{10}{9}$ to obtain a net width that can be compared with those lines that resist the full load. Use $w_n = 10.73(\frac{10}{9}) = 11.92$ inch. For line *abcdeg*,

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$w_n = 13.5 - 4(1) + \frac{(1.5)^2}{4(2.5)} + \frac{(1.5)^2}{4(4.75)} + \frac{(1.5)^2}{4(3)} = 10.03 \text{ in.}$$

The last case controls:

$$A_n = t(w_n) = 0.5(10.03) = 5.015 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.015 \text{ in.}^2$$

The design strength based on fracture is

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(5.015) = 218 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90 F_y A_g = 0.90(36)(6.75) = 219 \text{ kips}$$

ANSWER Fracture controls; design strength = 218 kips.

4.6 BLOCK SHEAR

- For some connection configurations, the tension member can fail due to 'tear-out' of material at the connected end. This is called *block shear*.
- For example, the single angle tension member connected as shown in the Figure below is susceptible to the phenomenon of *block shear*.

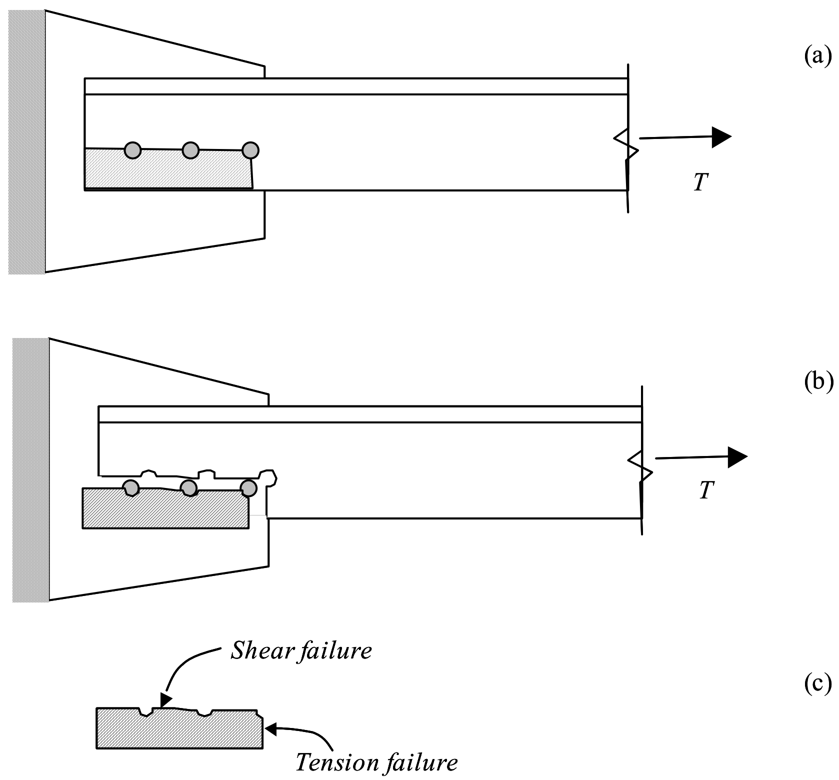


Figure 4.4 Block shear failure of single angle tension member

- For the case shown above, shear failure will occur along the longitudinal section a-b and tension failure will occur along the transverse section b-c
- AISC Specification (SPEC) Chapter D on tension members does not cover block shear failure explicitly. But, it directs the engineer to the Specification Section J4.3

- Block shear strength is determined as the <u>sum</u> of the shear strength on a failure path and the tensile strength on a perpendicular segment.
  - Block shear strength = net section fracture strength on shear path + gross yielding strength on the tension path
  - **<u>OR</u>**
  - Block shear strength = gross yielding strength of the shear path + net section fracture strength of the tension path
- Which of the two calculations above governs?
  - See page **16.1 – 67** (Section J4.3) of the AISC manual
  - When $F_u A_{nt} \geq 0.6 F_u A_{nv}$; $\phi_t R_n = \phi\,(0.6\ F_y A_{gv} + F_u A_{nt}) \leq \phi\,(0.6\ F_u A_{nv} + F_u A_{nt})$
  - When $F_u A_{nt} < 0.6 F_u A_{nv}$; $\phi_t R_n = \phi\,(0.6\ F_u A_{nv} + F_y A_{gt}) \leq \phi\,(0.6\ F_u A_{nv} + F_u A_{nt})$
  - Where, $\phi = 0.75$

    A_{gv} = gross area subject to shear

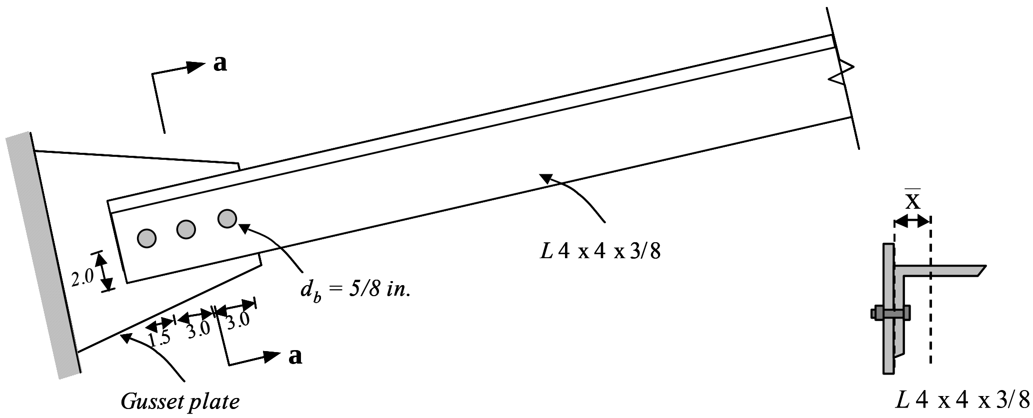
    A_{gt} = gross area subject to tension

    A_{nv} = net area subject to shear

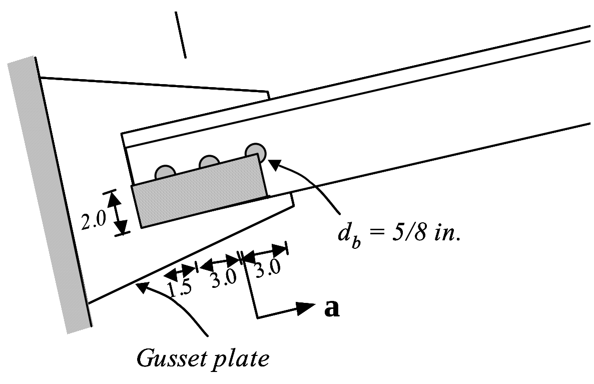
    A_{nt} = net area subject to tension

EXAMPLE 3.8 Calculate the block shear strength of the single angle tension member considered in Examples 3.2 and 3.3. The single angle *L 4 x 4 x 3/8* made from A36 steel is connected to the gusset plate with 5/8 in. diameter bolts as shown below. The bolt spacing is 3 in. center-to-center and the edge distances are 1.5 in and 2.0 in as shown in the Figure below.

Compare your results with those obtained in *Example 3.2 and 3.3*



- Step I. Assume a block shear path and calculate the required areas



- A_{gt} = gross tension area = 2.0 x 3/8 = 0.75 in^2
- A_{nt} = net tension area = 0.75 – **0.5** x (5/8 + 1/8) x 3/8 = 0.609 in^2
- A_{gv} = gross shear area = (3.0 + 3.0 +1.5) x 3/8 = 2.813 in^2
- A_{nv} = net tension area = 2.813 – **2.5** x (5/8 + 1/8) x 3/8 = 2.109 in^2

- Step II. Calculate which equation governs
  - $0.6\ F_u\ A_{nv} = 0.6 \times 58 \times 2.109 = 73.393$ kips
  - $F_u\ A_{nt} = 58 \times 0.609 = 35.322$ kips
  - $0.6\ F_u\ A_{nv} > F_u\ A_{nt}$
  - Therefore, equation with fracture of shear path governs
- Step III. Calculate block shear strength
  - $\phi_t\ R_n = 0.75\ (0.6\ F_u\ A_{nv} + F_y\ A_{gt})$
  - $\phi_t\ R_n = 0.75\ (73.393 + 36 \times 0.75) =$ **75.294 kips**
- Compare with results from previous examples

Example 3.2:

Ultimate factored load = P_u = 66 kips

Gross yielding design strength = $\phi_t\ P_n$ = 92.664 kips

Assume $A_e = 0.85\ A_n$

Net section fracture strength = 95.352 kips

Design strength = 92.664 kips (gross yielding governs)

Example 3.3

Calculate $A_e = 0.8166\ A_n$

Net section fracture strength = 91.045 kips

Design strength = 91.045 kips (net section fracture governs)

Member is still adequate to carry the factored load (P_u) = 66 kips

Example 3.8

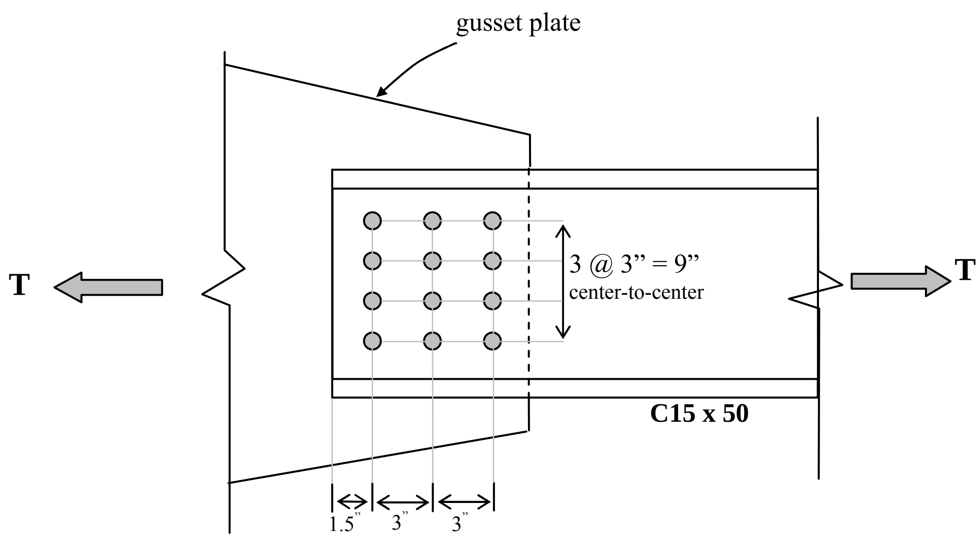
Block shear fracture strength = 75.294 kips

Design strength = 75.294 kips (block shear fracture governs)

Member is still adequate to carry the factored load (P_u) = 66 kips

- *Bottom line*:
  - Any of the three limit states (gross yielding, net section fracture, or block shear failure) can govern.
  - The design strength for all three limit states has to be calculated.
  - The member design strength will be the smallest of the three calculated values
  - The member design strength must be greater than the ultimate factored design load in tension.

Practice Example Determine the design tension strength for a single channel C15 x 50 connected to a 0.5 in. thick gusset plate as shown in Figure. Assume that the holes are for 3/4 in. diameter bolts and that the plate is made from structural steel with yield stress (F_y) equal to 50 ksi and ultimate stress (F_u) equal to 65 ksi.



- **Limit state of yielding due to tension:**

$$\phi T_n = 0.9 * 50 * 14.7 = 662 kips$$

- **Limit state of fracture due to tension:**

$$A_n = A_g - n d_e t = 14.7 - 4\left(\frac{7}{8}\right)(0.716) = 12.19 in^2$$

$$A_e = UA_n = \left(1 - \frac{x}{L}\right)A_n = \left(1 - \frac{0.798}{6}\right) * 12.19 = 10.57 in^2$$

Check: $U = 0.867 \leq 0.9$ OK.

Note: The connection eccentricity, x, for a C15X50 can be found on page 1-51 (LRFD).

$$\phi T_n = 0.75 * 65 * 10.57 = 515 kips$$

- **Limit state of block shear rupture:**

$$0.6 F_u A_{nv} = 0.6 * 65 * \left[2 * \left(7.5 - 2.5 * \frac{7}{8} \right) \right] * 0.716 = 296.6925$$

$$F_u A_{nt} = 65 * \left[9 - 3 \left(\frac{7}{8} \right) \right] * 0.716 = 296.6925$$

$$F_u A_{nt} \geq 0.6 F_u A_{nv}$$

$$\therefore \phi R_n = \phi \left[0.6 F_y A_{gv} + F_u A_{nt} \right] = 0.75 \left[0.6 * 50 * 15 * 0.716 + 65 * \frac{296.6925}{65} \right] = 464 kips$$

<u>Block shear rupture is the critical limit state and the design tension strength is 464kips.</u>

4.7 Design of tension members

- The design of a tension member involves finding the lightest steel section (angle, wide-flange, or channel section) with design strength (ϕP_n) greater than or equal to the maximum factored design tension load (P_u) acting on it.
  - $\phi P_n \geq P_u$
  - P_u is determined by structural analysis for factored load combinations
  - ϕP_n is the design strength based on the *gross section yielding*, *net section fracture*, and *block shear rupture* limit states.
- For gross yielding limit state, $\phi P_n = 0.9 \times A_g \times F_y$
  - Therefore, $0.9 \times A_g \times F_y \geq P_u$
  - Therefore, $A_g \geq \dfrac{P_u}{0.9 \times F_y}$
- For net section fracture limit state, $\phi P_n = 0.75 \times A_e \times F_u$
  - Therefore, $0.75 \times A_e \times F_u \geq P_u$
  - Therefore, $A_e \geq \dfrac{P_u}{0.75 \times F_u}$
  - But, $A_e = U\, A_n$
  - Where, U and A_n depend on the end connection.
- Thus, designing the tension member goes hand-in-hand with designing the end connection, which we have not covered so far.
- Therefore, for this chapter of the course, the end connection details will be given in the examples and problems.
- The AISC manual tabulates the tension design strength of standard steel sections
  - Include: wide flange shapes, angles, tee sections, and double angle sections.

- The gross yielding design strength and the net section fracture strength of each section is tabulated.
- This provides a great *starting point* for selecting a section.

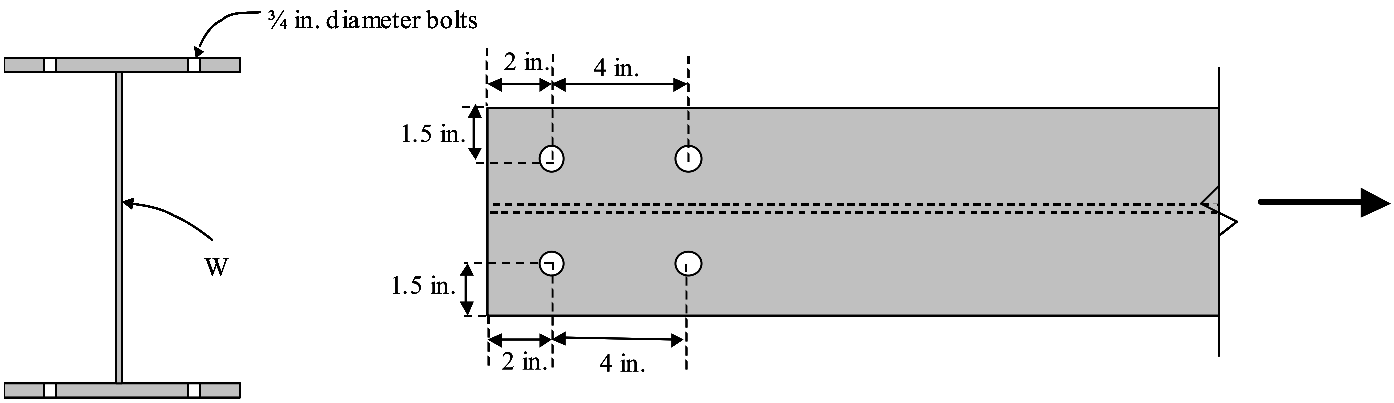
- **There is one serious limitation**
  - The net section fracture strength is tabulated for an assumed value of $U = 0.75$, obviously because the precise connection details are not known
  - For all W, Tee, angle and double-angle sections, A_e is assumed to be = 0.75 A_g
  - The engineer can **first** select the tension member based on the tabulated gross yielding and net section fracture strengths, and then check the net section fracture strength and the block shear strength using the actual connection details.

- Additionally for each shape the manual tells the value of A_e below which net section fracture will control:
  - Thus, for W shapes net section fracture will control if $A_e < 0.923\ A_g$
  - For single angles, net section fracture will control if $A_e < 0.745\ A_g$
  - For Tee shapes, net section fracture will control if $A_e < 0.923$
  - For double angle shapes, net section fracture will control if $A_e < 0.745\ A_g$

- **Slenderness limits**
  - Tension member slenderness l/r must preferably be limited to 300 as per LRFD specification B7

<u>Example 3.10</u> Design a member to carry a factored maximum tension load of 100 kips.

(a) Assume that the member is a wide flange connected through the flanges using eight ¾ in. diameter bolts in two rows of four each as shown in the figure below. The center-to-center distance of the bolts in the direction of loading is 4 in. The edge distances are 1.5 in. and 2.0 in. as shown in the figure below. Steel material is A992



Holes in beam flange

<u>SOLUTION</u>

- **Step I. Select a section from the Tables**
  - Go to the **TEN** section of the AISC manual. See Table 3-1 on pages 3-17 to 3-19.
  - From this table, select W8x10 with $A_g = 2.96$ in^2, $A_e = 2.22$ in^2.
  - Gross yielding strength = 133 kips, and net section fracture strength=108 kips
  - This is the lightest section in the table.
  - Assumed $U = 0.75$. And, net section fracture will govern if $A_e < 0.923\ A_g$
- **Step II. Calculate the net section fracture strength for the actual connection**
  - According to the Figure above, $A_n = A_g - 4\ (d_b + 1/8) \times t_f$
  - $A_n = 2.96 - 4\ (3/4 + 1/8) \times 0.205 = 2.24$ in^2
  - The connection is only through the flanges. Therefore, the shear lag factor U will be the distance from the top of the flange to the centroid of a WT 4 x 5.

- See **DIM** section of the AISC manual. See Table 1-8, on pages 1-50, 1-51
- $\bar{x}$ = 0.953
- $U = 1 - \bar{x}/L = 1 - 0.953 / 4 = 0.76$
- $A_e = 0.76\ A_n = 0.76 \times 2.24 = 1.70\ in^2$
- $\phi_t P_n = 0.75 \times F_u \times A_e = 0.75 \times 65 \times 1.70 = 82.9$ kips
- Unacceptable because P_u = 100 kips; **REDESIGN** required

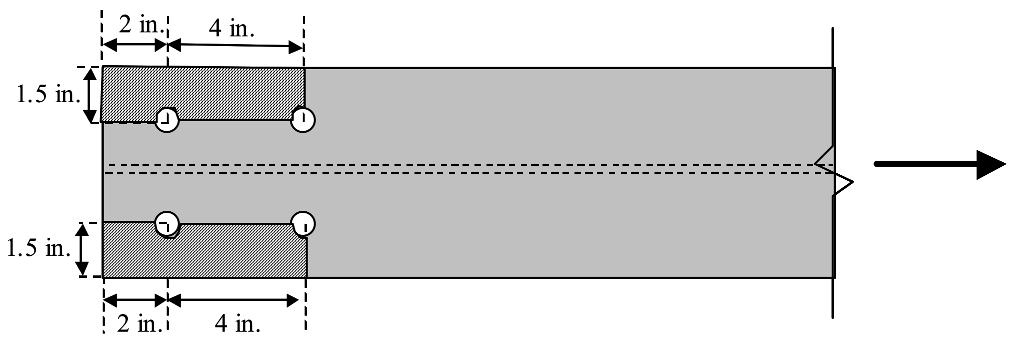
- **Step III. Redesign**

  Many ways to redesign. One way is shown here:

  - Assume $\phi_t P_n > 100$ kips
  - Therefore, $0.75 \times 65 \times A_e > 100$ kips
  - Therefore, $A_e > 2.051\ in^2$
  - Assume, $A_e = 0.76\ A_n$ (based on previous calculations, step II)
  - Therefore $A_n > 2.7\ in^2$
  - But, $A_g = A_n + 4\ (d_b + 1/8) \times t_f$ (based on previous calculations, step II)
  - Therefore $A_g > 2.7 + 3.5 \times t_f$
  - Go to the section dimension table 1-1 on page 1-22 of the AISC manual. Select next highest section.
    - For W 8 x 13, t_f = 0.255 in.
    - Therefore, $A_g > 2.7 + 3.5 \times 0.255 = 3.59\ in^2$
    - From Table 1-1, W8 x 13 has $A_g = 3.84\ in^2 > 3.59\ in^2$
    - Therefore, W8 x 13 is acceptable and is chosen.

- **Step IV. Check selected section for net section fracture**
  - $A_g = 3.84$ in^2
  - $A_n = 3.84 - 3.5 \times 0.255 = 2.95$ in^2
  - From dimensions of WT4 x 6.5, $\bar{x} = 1.03$ in.
  - Therefore, $U = 1 - \bar{x}/L = 1 - 1.03/4 = 0.74$
  - Therefore, $A_e = U\,A_n = 0.74 \times 2.95 = 2.19$ in^2
  - Therefore, net section fracture strength = 0.75 x 65 x 2.19 = 106.7 kips
  - Which is greater than 100 kips (design load). Therefore, W 8 x 13 is acceptable.

- **Step V. Check the block shear rupture strength**
  - o Identify the block shear path



  - The block shear path is show above. **<u>Four blocks</u>** will separate from the tension member (two from each flange) as shown in the figure above.
  - $A_{gv} = [(4+2) \times t_f\,] \times 4 \quad = 6 \times 0.255 \times 4 = 6.12$ in^2 - for four tabs
  - $A_{nv} = \{4+2 - 1.5 \times (d_b+1/8)\} \times t_f \times 4 = 4.78$ in^2
  - $A_{gt} = 1.5 \times t_f \times 4 = 1.53$ in^2
  - $A_{nt} = \{1.5 - 0.5 \times (d_b+1/8)\} \times t_f \times 4 = 1.084$ in^2
  - o Identify the governing equation:

- F_uA_{nt} = 65 x 1.084 = 70.4 kips
- $0.6F_uA_{nv}$ = 0.6 x 65 x 4.78 = 186.42 kips , which is > F_uA_{nt}

o Calculate block shear strength

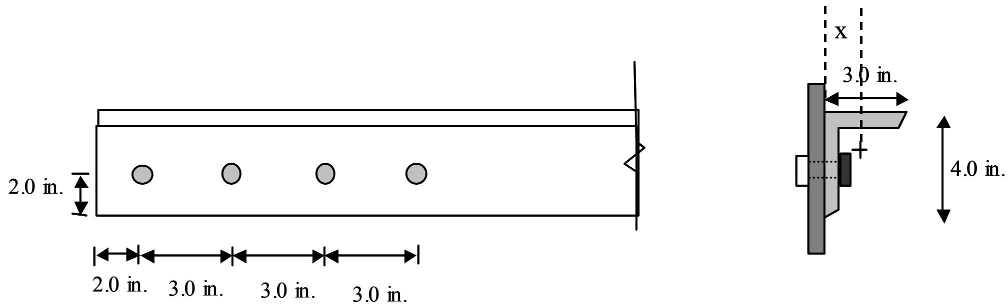
- $\phi_t R_n = 0.75\ (0.6F_uA_{nv} + F_yA_{gt})$ = 0.75 (186.42 + 50 x 1.53) = 197.2 kips
- Which is greater than P_u = 100 kips. Therefore W8 x 13 is still acceptable

- **Summary of solution**

Mem.	**Design load**	**A_g**	**A_n**	**U**	**A_e**	**Yield strength**	**Fracture strength**	**Block-shear strength**
W8x13	100 kips	3.84	2.95	0.74	2.19	173 kips	106.7 kips	197.2 kips
		Design strength = 106.7 kips (net section fracture governs) W8 x 13 is adequate for P_u = 100 kips and the given connection						

<u>EXAMPLE 3.11</u> Design a member to carry a factored maximum tension load of 100 kips.

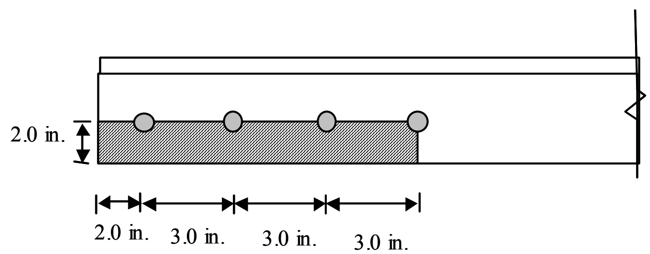
(b) The member is a single angle section connected through one leg using four 1 in. diameter bolts. The center-to-center distance of the bolts is 3 in. The edge distances are 2 in. Steel material is A36



- **Step I. Select a section from the Tables**
  - Go to the **TEN** section of the AISC manual. See Table 3-2 on pages 3-20 to 3-21.
  - From this table, select *L*4x3x1/2 with $A_g = 3.25$ in^2, $A_e = 2.44$ in^2.
  - Gross yielding strength = 105 kips, and net section fracture strength=106 kips
  - This is the lightest section in the table.
  - Assumed $U = 0.75$. And, net section fracture will govern if $A_e < 0.745\ A_g$
- **Step II. Calculate the net section fracture strength for the actual connection**
  - According to the Figure above, $A_n = A_g - 1\ (d_b + 1/8) \times t$
  - $A_n = 3.25 - 1(1 + 1/8) \times 0.5 = 2.6875$ in^2
  - The connection is only through the **<u>long leg</u>**. Therefore, the shear lag factor *U* will be the distance from the back of the long leg to the centroid of the angle.
  - See **DIM** section of the AISC manual. See Table 1-7, on pages 1-36, 1-37
  - $\bar{x} = 0.822$ in.
  - $U = 1 - \bar{x}/L = 1 - 0.822/9 = 0.908$
  - But *U* must be ≤ 0.90. Therefore, let $U = 0.90$

- $A_e = 0.90\ A_n = 0.90 \times 2.6875 = 2.41\ in^2$
- $\phi_t P_n = 0.75 \times F_u \times A_e = 0.75 \times 58 \times 2.41 = 104.8$ kips
- Acceptable because $P_u = 100$ kips.

- **Step V. Check the block shear rupture strength**
  - o Identify the block shear path



- $A_{gv} = (9+2) \times 0.5 = 5.5\ in^2$
- $A_{nv} = [11 - 3.5 \times (1+1/8)] \times 0.5 = 3.53\ in^2$
- $A_{gt} = 2.0 \times 0.5 = 1.0\ in^2$
- $A_{nt} = [2.0 - 0.5 \times (1 + 1/8)] \times 0.5 = 0.72\ in^2$

  - o Identify the governing equation:

- $F_u A_{nt} = 58 \times 0.72 = 41.76$ kips
- $0.6 F_u A_{nv} = 0.6 \times 58 \times 3.53 = 122.844\ in^2$, which is $> F_u A_{nt}$

  - o Calculate block shear strength

- $\phi_t R_n = 0.75\ (0.6 F_u A_{nv} + F_y A_{gt}) = 0.75\ (122.84 + 36 \times 1.0) = 119.133$ kips
- Which is greater than $P_u = 100$ kips. Therefore *L*4x3x1/2 is still acceptable

- **Summary of solution**

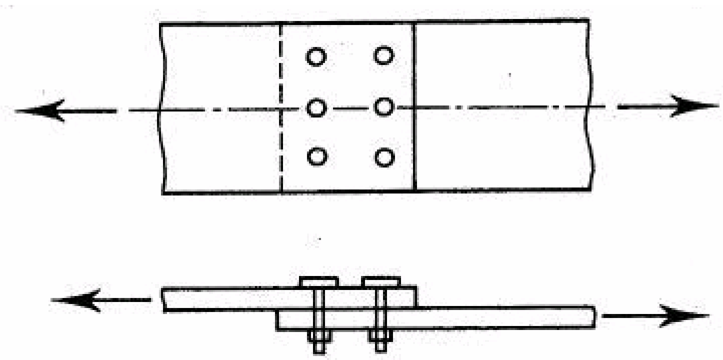
Mem.	**Design load**	**A_g**	**A_n**	**U**	**A_e**	**Yield strength**	**Fracture strength**	**Block-shear strength**
*L*4x3x1/2	100 kips	3.25	2.69	0.9	2.41	105 kips	104.8 kips	119.13 kips
		Design strength = 104.8 kips (net section fracture governs) *L*4x3x1/2 is adequate for P_u = 100 kips and the given connection						

- Note: For this problem A_e/A_g = 2.41/3.25 = 0.741, which is < 0.745. As predicted by the AISC manual, when A_e/A_g < 0.745, net section fracture governs.

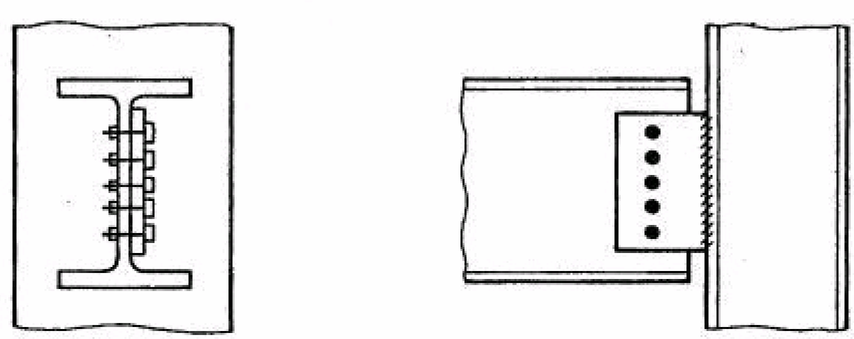
CHAPTER 5. BOLTED CONNECTION

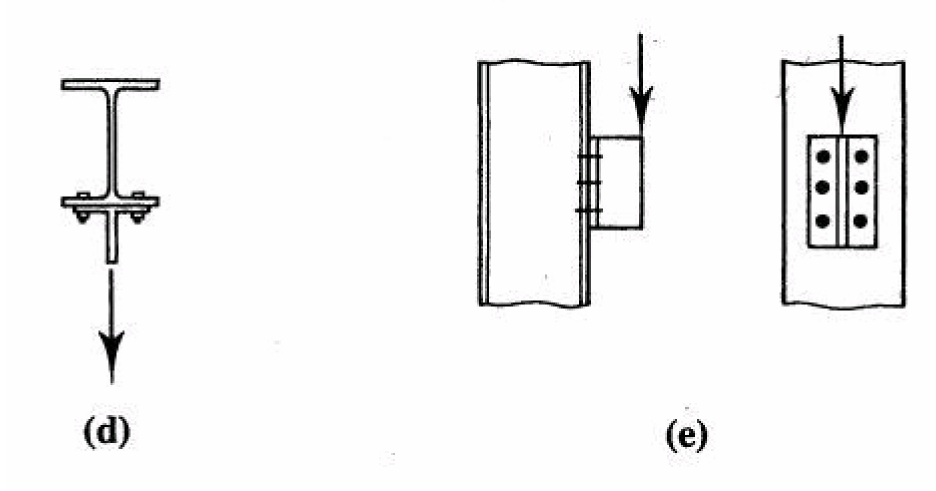
5.1 INTRODUCTORY CONCEPTS

- There are different types of bolted connections. They can be categorized based on the type of loading.
  - Tension member connection and splice. It subjects the bolts to forces that tend to shear the shank.
  - Beam end simple connection. It subjects the bolts to forces that tend to shear the shank.
  - Hanger connection. The hanger connection puts the bolts in tension



(a)

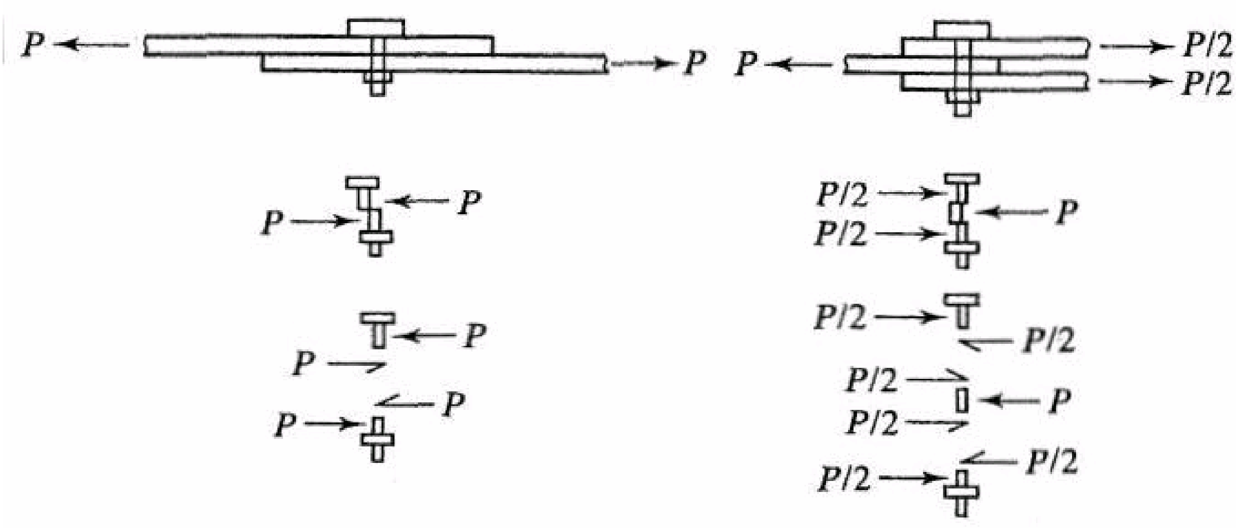




- The bolts are subjected to shear or tension loading.
  - In most bolted connection, the bolts are subjected to shear.
  - Bolts can fail in shear or in tension.
  - You can calculate the shear strength or the tensile strength of a bolt
- Simple connection: If the line of action of the force acting on the connection passes through the center of gravity of the connection, then each bolt can be assumed to resist an equal share of the load.
- The strength of the simple connection will be equal to the sum of the strengths of the individual bolts in the connection.
- We will first concentrate on bolted shear connections.

5.2 BOLTED SHEAR CONNECTIONS

- We want to design the bolted shear connections so that the factored design strength (ϕR_n) is greater than or equal to the factored load.
- So, we need to examine the various possible failure modes and calculate the corresponding design strengths.
- Possible failure modes are:
  - Shear failure of the bolts
  - Failure of member being connected due to fracture or block shear or ….
  - Edge tearing or fracture of the connected plate
  - Tearing or fracture of the connected plate between two bolt holes
  - Excessive bearing deformation at the bolt hole
- Shear failure of bolts
  - Average shearing stress in the bolt = $f_v = P/A = P/(\pi\ d_b^2/4)$
  - P is the load acting on an individual bolt
  - A is the area of the bolt and d_b is its diameter
  - Strength of the bolt = $P = f_v \times (\pi\ d_b^2/4)$ where f_v = shear yield stress = $0.6F_y$
  - Bolts can be in *single* shear or *double* shear as shown below.
  - When the bolt is in double shear, two cross-sections are effective in resisting the load. The bolt in *double shear* will have the twice the shear strength of a bolt in single shear.

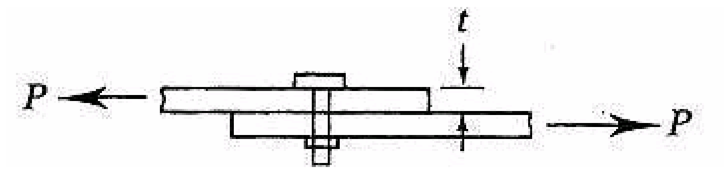


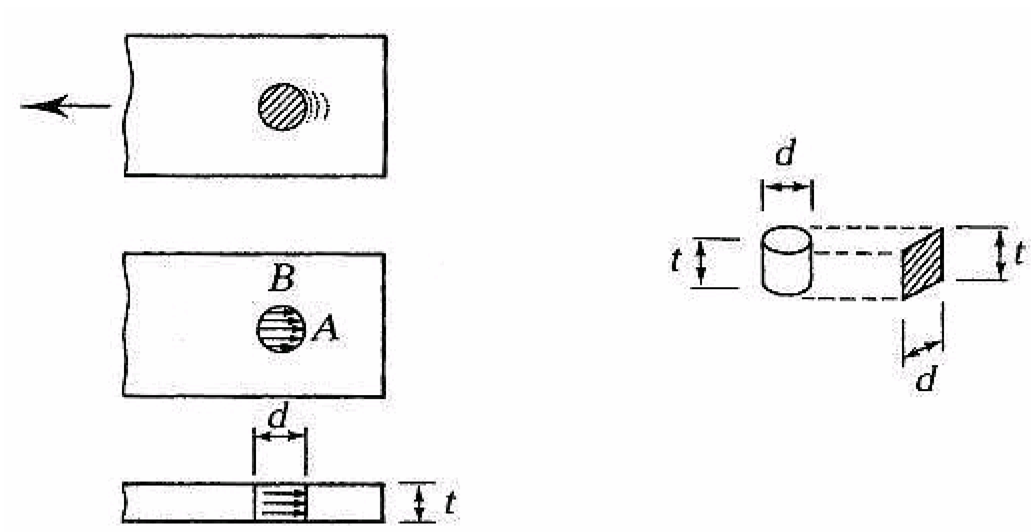
(a) Single Shear (b) Double Shear

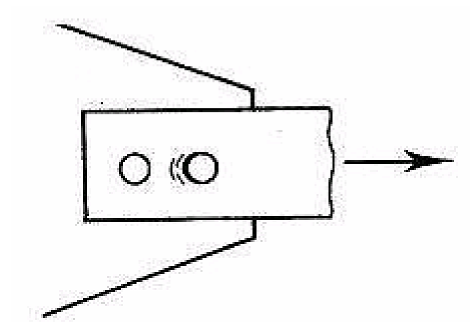
- Failure of connected member
  - We have covered this in detail in Ch. 2 on tension members
  - Member can fail due to tension fracture or block shear.
- Bearing failure of connected/connecting part due to bearing from bolt holes
  - Hole is slightly larger than the fastener and the fastener is loosely placed in hole
  - Contact between the fastener and the connected part over approximately half the circumference of the fastener
  - As such the stress will be highest at the radial contact point (A). However, the average stress can be calculated as the applied force divided by the projected area of contact
  - Average bearing stress $f_p = P/(d_b\, t)$, where P is the force applied to the fastener.
  - The bearing stress state can be complicated by the presence of nearby bolt or edge. The bolt spacing and edge distance will have an effect on the bearing str.
  - Bearing stress effects are independent of the bolt type because the bearing stress acts on the connected plate not the bolt.

- A possible failure mode resulting from excessive bearing close to the edge of the connected element is shear tear-out as shown below. This type of shear tear-out can also occur between two holes in the direction of the bearing load.

  $R_n = 2 \times 0.6\ F_u\ L_c\ t = 1.2\ F_u\ L_c\ t$







- To prevent excessive deformation of the hole, an upper limit is placed on the bearing load. This upper limit is proportional to the fracture stress times the projected bearing area

  $R_n = C \times F_u \times$ bearing area $= C\ F_u\ d_b\ t$

  If deformation is not a concern then $C = 3$, If deformation is a concern then $C=2.4$

  $C = 2.4$ corresponds to a deformation of 0.25 in.

- Finally, the equation for the bearing strength of a single bolts is ϕR_n

  where, $\phi = 0.75$ and $R_n = 1.2\ L_c\ t\ F_u < 2.4\ d_b\ t\ F_u$

  L_c is the clear distance in the load direction, from the edge of the bolt hole to the edge of the adjacent hole or to the edge of the material

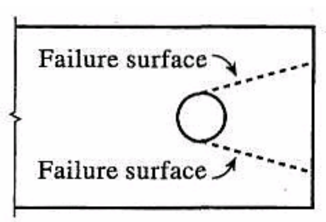
- This relationship can be simplified as follows:

  The upper limit will become effective when $1.2\ L_c\ t\ F_u = 2.4\ d_b\ t\ F_u$

  i.e., the upper limit will become effective when $L_c = 2\ d_b$

  If $L_c < 2\ d_b$, $R_n = 1.2\ L_c\ t\ F_u$

  If $L_c > 2\ d_b$, $R_n = 1.4\ d_b\ t\ F_u$



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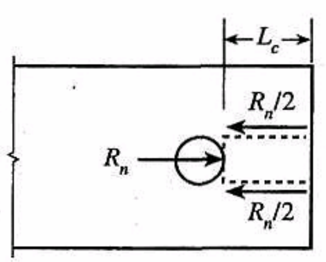
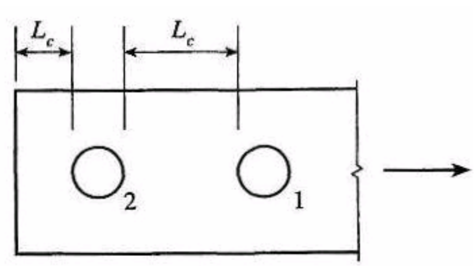
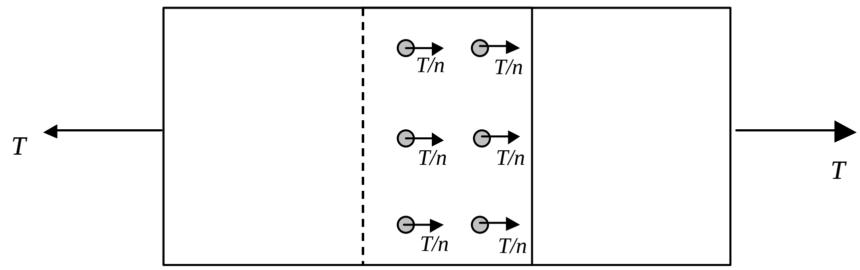


(b)

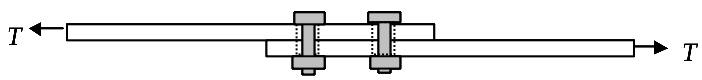


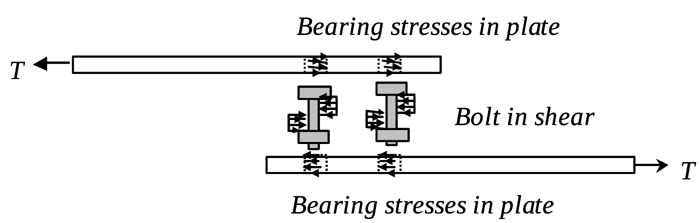
5.3 DESIGN PROVISIONS FOR BOLTED SHEAR CONNECTIONS

- In a simple connection, all bolts share the load equally.



- In a bolted shear connection, *the bolts are subjected to shear* and the connecting / connected plates are subjected to bearing stresses.





- The shear strength of all bolts = shear strength of one bolt x number of bolts
- The bearing strength of the connecting / connected plates can be calculated using equations given by AISC specifications.
- The tension strength of the connecting / connected plates can be calculated as discussed earlier in Chapter 2.

5.3.1 AISC Design Provisions

- Chapter J of the AISC Specifications focuses on connections.
- Section J3 focuses on bolts and threaded parts

- AISC Specification J3.3 indicates that the minimum distance (s) between the centers of bolt holes is $2\frac{2}{3}d_b$. A distance of $3d_b$ is preferred.
- AISC Specification J3.4 indicates that the minimum edge distance (L_e) from the center of the bolt to the edge of the connected part is given in Table J3.4 on page **16.1**-61. Table J3.4 specifies minimum edge distances for sheared edges, edges of rolled shapes, and gas cut edges.
- AISC Specification J3.5 indicates that the maximum edge distance for bolt holes is 12 times the thickness of the connected part (but not more than 6 in.). The maximum spacing for bolt holes is 24 times the thickness of the thinner part (but not more than 12 in.).
- Specification J3.6 indicates that the design tension or shear strength of bolts is $\phi F_n A_b$
  - Table J3.2 gives the values of ϕ and F_n
  - A_b is the unthreaded area of bolt.
  - In Table J3.2, there are different types of bolts A325 and A490.
  - The shear strength of the bolts depends on whether threads are included or excluded from the shear planes. If threads are included in the shear planes then the strength is lower.
  - We will always assume that threads are included in the shear plane, therefore less strength to be conservative.
- We will look at specifications J3.7 – J3.9 later.
- AISC Specification J3.10 indicates the bearing strength of plates at bolt holes.
  - The design bearing strength at bolt holes is ϕR_n
  - $R_n = 1.2\ L_c\ t\ F_u \leq 2.4\ d_b\ t\ F_u$ - deformation at the bolt holes is a design consideration
  - Where, F_u = specified tensile strength of the connected material

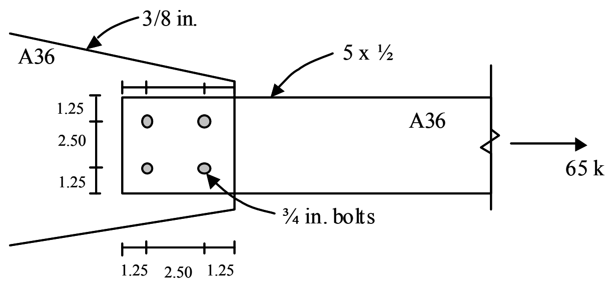
- L_c = clear distance, in the direction of the force, between the *edge* of the hole and the *edge* of the adjacent hole or edge of the material (in.).
- t = thickness of connected material

5.3.2 AISC Design Tables

- Table 7-10 on page 7-33 of the AISC Manual gives the design shear of one bolt. Different bolt types (A325, A490), thread condition (included or excluded), loading type (single shear or double shear), and bolt diameters (5/8 in. to 1-1/2 in.) are included in the Table.
- Table 7-11 on page 7-33 of the AISC Manual is an extension of Table 7-10 with the exception that it gives the shear strength of '*n*' bolts.
- Table 7-12 on page 7-34 of the AISC manual gives the design bearing strength at bolt holes for various bolt spacings.
  - These design bearing strengths are in kips/in. thickness.
  - The tabulated numbers must be multiplied by the plate thickness to calculate the design bearing strength of the plate.
  - The design bearing strengths are given for different bolt spacings ($2.67d_b$ and $3d_b$), different F_u (58 and 65 ksi), and different bolt diameters (5/8 – 1-1/2 in.)
  - Table 7-12 also includes the spacing (s_{full}) required to develop the full bearing strength for different F_u and bolt diameters
  - Table 7-12 also includes the bearing strength when $s > s_{full}$
  - Table 7-12 also includes the minimum spacing 2-2/3 d_b values
- Table 7-13 in the AISC manual on page 7-35 is similar to Table 7-12. It gives the design bearing strength at bolt holes for various edge distances.

- These design bearing strengths are in *kips/in. thickness*.
- The tabulated numbers must be multiplied by the plate thickness to calculate the design bearing strength of the plate.
- The design bearing strengths are given for different edge distances (1.25 in. and 2 in.), different F_u (58 and 65 ksi), and different bolt diameters (5/8 – 1-1/2 in.)
- Table 7-13 also includes the edge distance ($L_{e\ full}$) required to develop the full bearing strength for different F_u and bolt diameters
- Table 7-13 also includes the bearing strength when $L_e > L_{e\ full}$

Example 5.1 Calculate and check the design strength of the connection shown below. Is the connection adequate for carrying the factored load of 65 kips.



Solution

Step I. Shear strength of bolts

- The design shear strength of one bolt in shear = $\phi F_n A_b = 0.75 \times 48 \times \pi \times 0.75^2/4$
  - $\phi F_n A_b$ = 15.9 kips per bolt (See Table J3.2 and Table 7-10)
  - Shear strength of connection = 4 x 15.9 = 63.6 kips (See Table 7-11)

Step II. Minimum edge distance and spacing requirements

- See Table J3.4, minimum edge distance = 1 in. for rolled edges of plates
  - The given edge distances (1.25 in.) > 1 in. Therefore, minimum edge distance requirements are satisfied.
- Minimum spacing = $2.67\, d_b = 2.67 \times 0.75 = 2.0$ in.
  - Preferred spacing = $3.0\, d_b = 3.0 \times 0.75 = 2.25$ in.
  - The given spacing (2.5 in.) > 2.25 in. Therefore, spacing requirements are satisfied.

Step III. Bearing strength at bolt holes.

- Bearing strength at bolt holes in connected part (5 x ½ in. plate)
  - At edges, L_c = 1.25 – hole diameter/2 = 1.25 – (3/4 + 1/16)/2 = 0.844 in.
  - $\phi R_n = 0.75 \times (1.2\, L_c\, t\, F_u) = 0.75 \times (1.2 \times 0.844 \times 0.5 \times 58) = 22.02$ kips
  - But, $\phi R_n \leq 0.75\, (2.4\, d_b\, t\, F_u) = 0.75 \times (2.4 \times 0.75 \times 0.5 \times 58) = 39.15$ kips
  - Therefore, ϕR_n = 22.02 kips at edge holes

    - *Compare with value in Table 7-13. $\phi R_n = 44.0 \times 0.5 = 22.0$ kips*

- At other holes, s = 2.5 in, L_c = 2.5 – (3/4 +1/16) = 1.688 in.
- ϕR_n = 0.75 x (1.2 L_c t F_u) = 0.75 x (1.2 x 1.688 x 0.5 x 58) = 44.05 kips
- But, $\phi R_n \leq$ 0.75 (2.4 d_b t F_u) = 39.15 kips. Therefore ϕR_n = 39.15 kips
- Therefore, ϕR_n = 39.15 kips at other holes
    - *Compare with value in Table 7-12. ϕR_n = 78.3 x 0.5 =39.15 kips*
- Therefore, bearing strength at holes = 2 x 22.02 + 2 x 39.15 = 122.34 kips

- Bearing strength at bolt holes in gusset plate (3/8 in. plate)
    - At edges, L_c = 1.25 – hole diameter/2 = 1.25 – (3/4 + 1/16)/2 = 0.844 in.
    - ϕR_n = 0.75 x (1.2 L_c $\underline{t}$ F_u) = 0.75 x (1.2 x 0.844 x <u>0.375</u> x 58) = 16.52 k
    - But, $\phi R_n \leq$ 0.75 (2.4 d_b $\underline{t}$ F_u) = 0.75 x (2.4 x 0.75 x <u>0.375</u> x 58) = 29.36 kips
    - Therefore, ϕR_n = 16.52 kips at edge holes
        - *Compare with value in Table 7-13. ϕR_n = 44.0 x 3/8 = 16.5 kips*
    - At other holes, s = 2.5 in, L_c = 2.5 – (3/4 +1/16) = 1.688 in.
    - ϕR_n = 0.75 x (1.2 L_c $\underline{t}$ F_u) = 0.75 x (1.2 x 1.688 x <u>0.375</u> x 58) = 33.04 kips
    - But, $\phi R_n \leq$ 0.75 (2.4 d_b t F_u) = 29.36 kips
    - Therefore, ϕR_n = 29.36 kips at other holes
        - *Compare with value in Table 7-12. ϕR_n = 78.3 x 0.375 = 29.36 kips*
    - Therefore, bearing strength at holes = 2 x 16.52 + 2 x 29.36 = 91.76 kips

- Bearing strength of the connection is the smaller of the bearing strengths = 91.76 kips

Connection Strength
Shear strength = 63.3 kips
Bearing strength (plate) = 122.34 kips
Bearing strength (gusset) = 91.76 kips

Connection strength (ϕR_n) > applied factored loads (γQ). Therefore ok.

Example 5.2 Design a double angle tension member and a gusset plated bolted connection system to carry a factored load of 100 kips. Assume A36 (36 ksi yield stress) material for the double angles and the gusset plate. Assume A325 bolts. Note that you have to design the double angle member sizes, the gusset plate thickness, the bolt diameter, numbers, and spacing.

Solution

Step I. Design and select a trial tension member

- See **Table 3-7** on page 3-33 of the AISC manual.
  - Select 2*L* 3 x 2 x 3/8 with ϕP_n = 113 kips (yielding) and 114 kips (fracture)
  - While selecting a trial tension member check the fracture strength with the load.

Step II. Select size and number of bolts

The bolts are in double shear for this design (may not be so for other designs)

- See **Table 7-11** on page 7-33 in the AISC manual

  Use four 3/4 in. A325 bolts in double shear

  ϕR_n = 127 kips - shear strength of bolts from Table 7-11

Step III. Design edge distance and bolt spacing

- **See Table J3.4**
  - The minimum edge distance = 1 in. for 3/4 in. diameter bolts in rolled edges.
  - Select edge distance = 1.25 in.
- **See specification J3.5**
  - Minimum spacing = 2.67 d_b = 2.0 in.
  - Preferred spacing = 3.0 d_b = 2.25 in.
  - Select spacing = 3.0 in., which is greater than preferred or minimum spacing

Step IV. Check the bearing strength at bolt holes in angles

- Bearing strength at bolt holes in angles
  - Angle thickness = 3/8 in.
  - See **Table 7-13** for the bearing strength per in. thickness at the edge holes
  - Bearing strength at the edge holes (L_e = 1.25 in.) = ϕR_n = 44.0 x 3/8 = 16.5 k
  - See **Table 7-12** for the bearing strength per in. thickness at non-edge holes
  - Bearing strength at non-edge holes (s = 3 in.) = ϕR_n = 78.3 x 3/8 = 29.4 k
  - *Bearing strength at bolt holes in each angle = 16.5 + 3 x 29.4 = 104.7 kips*
  - *Bearing strength of double angles = 2 x 104.7 kips = 209.4 kips*

Step V. Check the fracture and block shear strength of the tension member

- *This has been covered in the chapter on tension members and is left to the students.*

Step VI. Design the gusset plate

- See **specification J5.2** for designing gusset plates. These plates must be designed for the limit states of yielding and rupture
  - Limit state of yielding
    - $\phi R_n = 0.9\ A_g\ F_y > 100$ kips
    - Therefore, $A_g = L \times t > 3.09\ in^2$
    - Assume t = ½ in; Therefore L > 6.18 in.
    - Design gusset plate = 6.5 x ½ in.
    - Yield strength = ϕR_n = 0.9 x 6.5 x 0.5 x 36 = 105.3 kips
  - Limit state for fracture

- o $A_n = A_g - (d_b + 1/8) \times t$
- o $A_n = 6.5 \times 0.5 - (3/4 + 1/8) \times 0.5 = 2.81\ in^2$
- o **But, $A_n \leq 0.85\ A_g = 0.85 \times 3.25 = 2.76\ in^2$**
- o $\phi R_n = 0.75 \times A_n \times F_u = 0.75 \times$ **2.76** $\times 58 = 120$ kips

- Design gusset plate = 6.5 x 0.5 in.

• **Step VII.** Bearing strength at bolt holes in gusset plates

Assume L_e = 1.25 in. (same as double angles)

- Plate thickness = 3/8 in.
- Bearing strength at the edge holes = ϕR_n = 44.0 x 1/2 = 22.0 k
- Bearing strength at non-edge holes = ϕR_n = 78.3 x 1/2 = 39.15 k
- *Bearing strength at bolt holes in gusset plate = 22.0 + 3 x 39.15 = 139.5 kips*

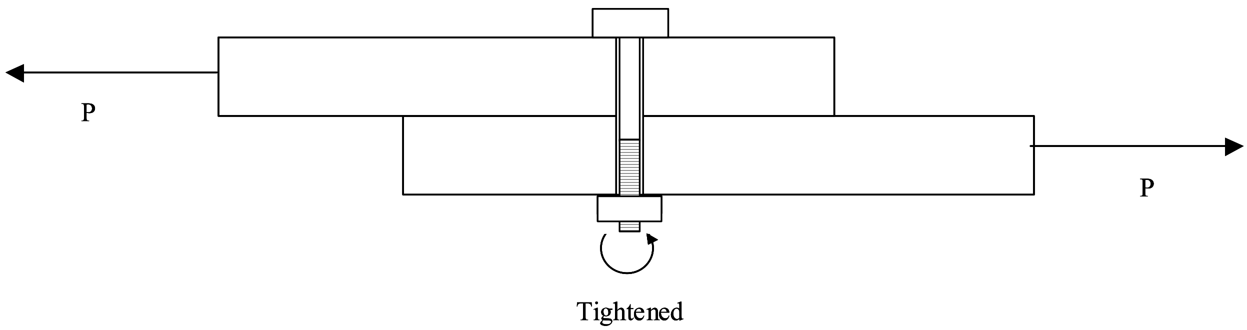
Summary of Member and Connection Strength

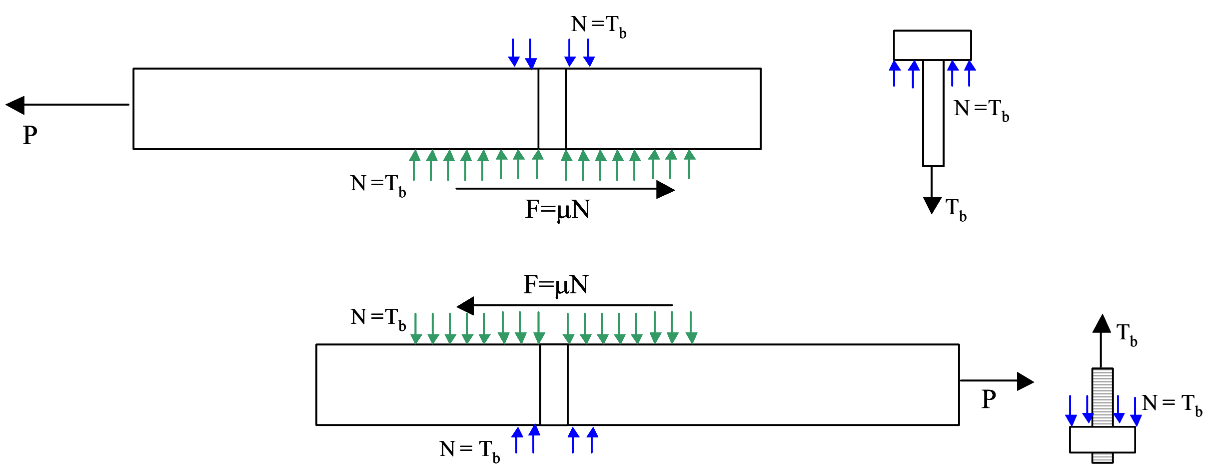
Connection	Member	Gusset Plate
Shear strength = 127 kips	Yielding = 113 kips	Yielding = 105.3 kips
Bearing strength = 209.4 kips (angles)	Fracture = ?	Fracture = 120 kips
Bearing Strength = 139.5 (gusset)	Block Shear = ?	

- Overall Strength is the smallest of all these numbers = 105.3 kips
- Gusset plate yielding controls
- Resistance > Factored Load (100 kips).
- Design is acceptable

5.4 SLIP-CRITICAL BOLTED CONNECTIONS

- High strength (A325 and A490) bolts can be installed with such a degree of tightness that they are subject to large tensile forces.
- These large tensile forces in the bolt clamp the connected plates together. The shear force applied to such a tightened connection will be resisted by friction as shown in the Figure below.





- Thus, *slip-critical bolted connections* can be designed to resist the applied shear forces using friction. If the applied shear force is less than the friction that develops between the two surfaces, then no slip will occur between them.
- However, slip will occur when the friction force is less than the applied shear force. After slip occurs, the connection will behave similar to the bearing-type bolted connections designed earlier.

- Table J3.1 summarizes the minimum bolt tension that must be applied to develop a slip-critical connection.
- The shear resistance of fully tensioned bolts to slip <u>at factored loads</u> is given by AISC Specification J3.8 a

  Shear resistance at factored load = $\phi R_n = 1.13\ \mu\ T_b\ N_s$

  where, ϕ = 1.0 for standard holes

  μ = 0.33 (Class A surface with unpainted clean mill scale surface: CE 405)

  T_b = minimum bolt tension given in Table J3.1

  N_s = number of slip planes

  - See Table 7-15 on page 7-36 of the AISC manual. This Table gives the shear resistance of fully tensioned bolts to slip at factored loads on class A surfaces.
  - For example, the shear resistance of 1-1/8 in. bolt fully tensioned to 56 kips (Table J3.1) is equal to 20.9 kips (Class A faying surface).
  - When the applied shear force exceeds the ϕR_n value stated above, slip will occur in the connection.
- The shear resistance of fully tensioned bolts to slip <u>at service loads</u> is given by AISC Specification J3.8 b.
  - Shear resistance at service load = $\phi R_n = \phi\ F_v\ A_b$
  - Where, ϕ = 1.0 for standard holes

    F_v = slip-critical resistance to shear at service loads, see **Table A-J3.2** on page 16.1-116 of the AISC manual
  - See Table 7-16 on page 7-37 of the AISC manual. This Table gives the shear resistance of fully tensioned bolts to slip at service loads on class A surfaces.

- For example, the shear resistance of 1-1/8 in. bolt fully tensioned to 56 kips (Table J3.1) is equal to 16.9 kips (Class A faying surface).
- When the applied shear force exceeds the ϕR_n value stated above, slip will occur in the connection.

• The final strength of the connection will depend on the shear strength of the bolts calculated using the values in Table 7-11 and on the bearing strength of the bolts calculated using the values in Table 7-12, 7-13. This is the same strength as that of a bearing type connection.

Example 5.3 Design a slip-critical splice for a tension member subjected to 300 kips of tension loading. The tension member is a W8 x 28 section made from A992 (50 ksi) material. The unfactored dead load is equal to 50 kips and the unfactored live load is equal to 150 kips. Use A325 bolts. The splice should be slip-critical at service loads.

Solution

Step I. Service and factored loads

- Service Load = D + L = 200 kips.
- Factored design load = 1.2 D + 1.6 L = 300 kips
- Tension member is W8 x 28 section made from A992 (50 ksi) steel. The tension splice must be slip critical (i.e., it must not slip) at service loads.

Step II. Slip-critical splice connection

- ϕR_n of one fully-tensioned slip-critical bolt = $\phi\ F_v\ A_b$ (See Spec. **A-J3.8 b**) *page 16.1-117 of AISC*
- If d_b = 3/4 in.

  ϕR_n of one bolt = $1.0 \times 17 \times \pi \times 0.75^2/4 = 7.51$ kips

  Note, F_v = 17 ksi from Table A-J3.2

  From **Table 7-16** on page **7-37** ϕRn = 7.51 kips

  ϕR_n of n bolts = 7.51 x n > 200 kips (splice must be slip-critical at service)

  Therefore, n > 26.63
- If d_b = 7/8 in.

  ϕR_n of one bolt = 10.2 kips -from **Table 7-16**

  ϕR_n of n bolts = 10.2 x n > 200 kips (splice must be slip-critical at service)

  Therefore, n > 19.6 bolts

Example 5.3

Step I: Service and Factored Loads

$D := 50$ Kips $\quad L := 150$ Kips

- Service Loads $\quad P_s := D + L \quad\quad P_s = 200$ Kips
- Factored Loads $\quad P_u := 1.2 \cdot D + 1.6 \cdot L \quad\quad P_u = 300$ Kips

Step II: Slip Critical connection

- In Service loads consideration, ϕR_n of one fully tenstioned slip-critical bolt = $\phi\, F_v\, A_b$

  (As given in Spec. A-J3.8b - page 16.1-117) $\quad \phi := 1.0 \quad F_v := 17$ Ksi - A325 - Table A-J3.2

- If diameter of the bolt = $d_b := \frac{3}{4}$ in $\quad A_b := \frac{\pi}{4}\left(d_b\right)^2$

  for one bolt $\quad \phi R_n := \phi \cdot F_v \cdot A_b \quad\quad \phi R_n = 7.51$ Kips

  Number of bolts required $\quad n := \frac{P_s}{\phi R_n} \quad\quad n = 26.63$ (min. reqd.)

- If diameter of the bolt = $d_b := \frac{7}{8}$ in $\quad A_b := \frac{\pi}{4}\left(d_b\right)^2$

  for one bolt $\quad \phi R_n := \phi \cdot F_v \cdot A_b \quad\quad \phi R_n = 10.22$: Kips

  Number of bolts required $\quad n := \frac{P_s}{\phi R_n} \quad\quad n = 19.565$ (min. reqd.)

  say we provide **24 bolts** on either side of the center line, 6 on either side of the flanges, top + bottom

Step III: Connection Details and spacings for 24 bolts on each W8 x 28

- Note that there are 24 bolts on either side of the center line. In all there are 48 number - 7/8 in dia bolts used in the connection.

- Minimum pretension applied to the bolts = 39.0 Kips from Table J3.1

- Minimum Edge distance from Table J3.4 = $L_{e\text{-}min}$ = 1.125 in

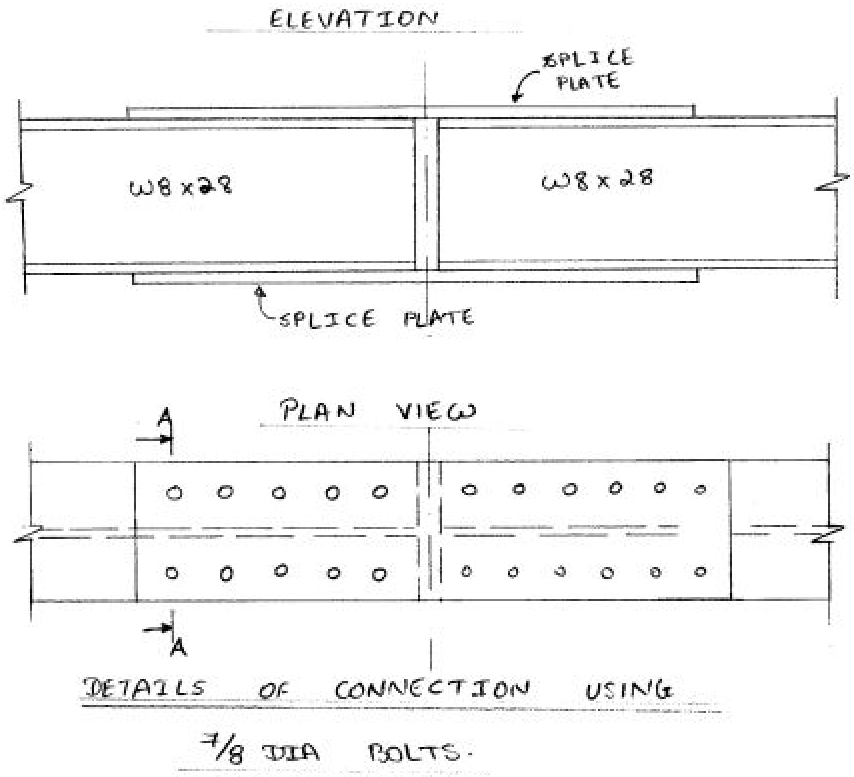
- Provide Edge Distance = $L_e := 1.25$ in

- Minimum spacing (Spec. J3.3) = $s := 2.67 \cdot d_b \quad\quad s = 2.336$ in

Preferred spacing = $s := 3 \cdot d_b$ $s = 2.625$ in

From table 7-12 $s_{full} := 2.6875$ in

For design provide spacing = $s := 3$ in



Step IV: Connection Strength at factored loads

- The splice connection should be designed as a normal shear / bearing connection beyong this point for the factored load = 300 kips
- The shear strength of the bolts (Table 7-10) = 21.6 kips/bolt x 24 bolts = 518.4 Kips
- Bearing strength of 7/8 in. bolt at edge holes = $B_e := 45.7$ Kip / in thickness Table 7-13
- Bearing strength of 7/8 in. bolt at other holes = $B_o := 102$ Kip / in thickness Table 7-12
- Total bearing strength of the bolt holes in wide flange section $B_t := 4 \cdot B_e + 20 \cdot B_o$

$B_t = 2.223 \times 10^3$ Kips

Step V: Design the splice plate $F_y := 50$ Ksi $F_u := 65$ Ksi $P_u = 300$ Kips

- Tension Yielding = 0.9 $A_g F_y > P_u$ $\quad minA_g := \dfrac{P_u}{0.9 \cdot F_y}$ $\quad minA_g = 6.667 \ in^2$

- Tension Fracture = 0.75 $A_n F_u > P_u$ $\quad minA_n := \dfrac{P_u}{0.75 \cdot F_u}$ $\quad minA_n = 6.154 \ in^2$

We know, flange width of W 8 x 28 = 6.54 in. This is the limiting width of the splice plate. The unknown quantity which is the thickness of each splice plate is calculated as shown.

Net area = Gross area - area of the bolts $\quad A_n := minA_n$

$A_n := A_g - 4 \cdot \left(\frac{7}{8} + \frac{1}{8}\right) \cdot t$ $\quad$ Here, $A_g := 6.54 \cdot t$ $\quad A_n := 6.154 \ in^2$

Solving for t, we get $\quad t_{min} := 2.42 \ in$ $\quad$ (This is the total thickness of the plate at the top and bottom)

Assume each plate of the dimensions $\quad b := 6.54 \ in \quad t_p := 1.25 \ in$

therefore, total plate thickness = $\quad t := 2 \cdot t_p \quad t = 2.5 \ in \quad > \quad t_{min} = 2.42 \ in$

Check for A_n and A_g:

$A_g := b \cdot t \quad A_g = 16.35 \ in^2 \quad > \quad minA_g = 6.667 \ in^2$

$A_n := A_g - 4 \cdot \left(\frac{7}{8} + \frac{1}{8}\right) \cdot t \quad A_n = 6.35 \ in^2 \quad > \quad minA_n = 6.154 \ in^2$

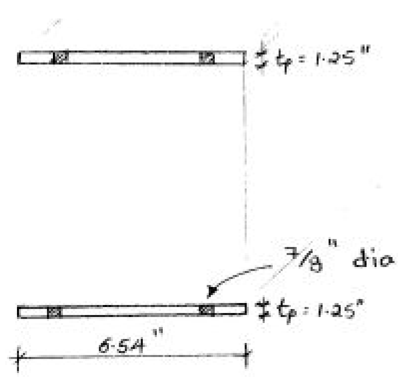
Check $\quad A_n = 6.35 \ in^2 \quad < \quad 0.85 \cdot A_g = 13.898 \ in^2$

- Strength of the splice plate in

  yielding = $0.9 \cdot A_g \cdot F_y = 735.75$ Kips

  fracture = $0.75 \cdot A_n \cdot F_u = 309.563$ Kips

  $> \quad P_u = 300$ Kips



- Check for bearing strength of the splice plates

- Check for block shear rupture

<u>Step VI : Check member yield, fracture and block shear....</u>

Example 4.4 Modify Example 4.2 so that the connection system is slip critical for the factored load of 100 kips.

Solution

Step I. Design and select a trial tension member (same as **example 4.2**)

- Select 2*L* 3 x 2 x 3/8 with ϕP_n = 113 kips (yielding) and 114 kips (fracture)

Step II. Select size and number of bolts (modified step)

- The connection must be designed to be slip-critical at the factored loads
  - ϕR_n for one bolt = 1.0 x 1.13 x μ x T_b x N_s (T_b from **Table J3.1**)
  - ϕR_n for one 3/4 in. bolt = 1.0 x 1.13 x 0.33 x 28 x 2 = 20.9 kips
  - ϕR_n for one 7/8 in. bolt = 1.0 x 1.13 x 0.33 x 39 x 2 = 29.1 kips
  - **See Values in Table 7-15**.

    ϕR_n for ¾ and 7/8 in. bolts in double slip = 20.9 and 29.1 kips, respectively.
  - We need at least five ¾ in. bolts to have strength ϕR_n = 5 x 20.9 = 104.5 k > 100 k
  - We need at least four 7/8 in. bolts to have strength ϕR_n = 4 x 29.1 = 116.4 k> 100
- **Use five ¾ in. fully tightened bolts**. Bolts must be tightened to 28 kips.
- Compare with solution for example 4.2 where only four snug-tight ¾ in bolts design.

For the remaining steps III to VII follow Example 4.2

Step III. Design edge distance and bolt spacing

Step IV. Check the bearing strength at bolt holes in angles

Step V. Check the fracture and block shear strength of the tension member

Step VI. Design the gusset plate

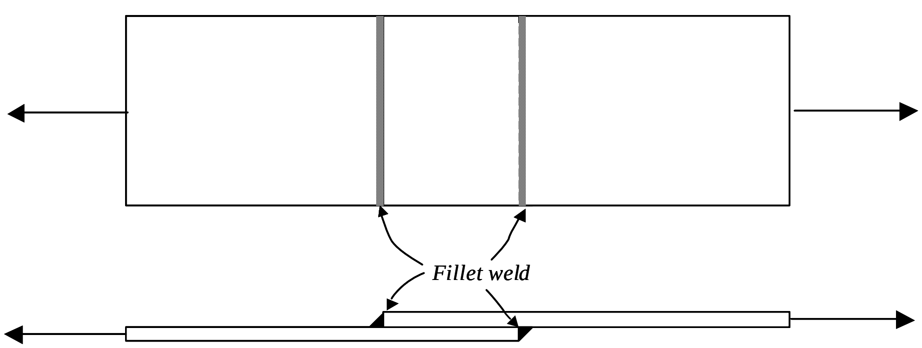
Step VII. Bearing strength at bolt holes in gusset plates

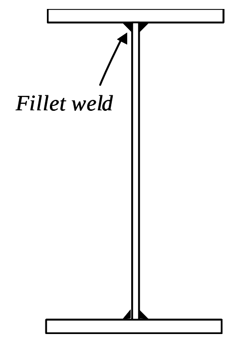
Summary of Member and Connection Strength

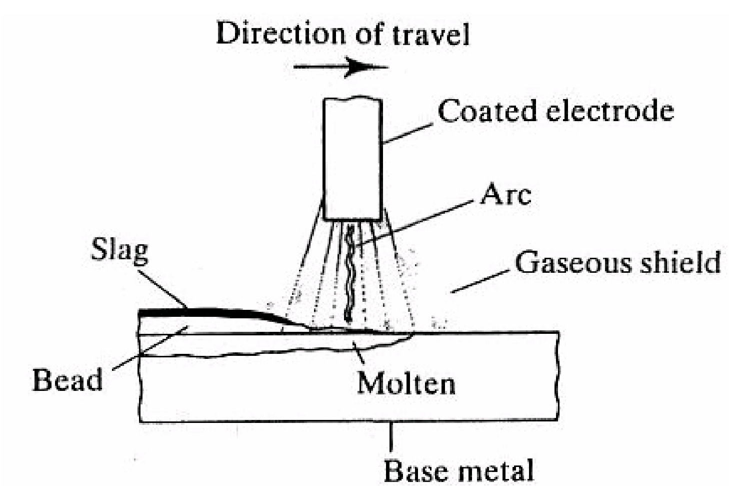
CHAPTER 6. WELDED CONNECTIONS

6.1 INTRODUCTORY CONCEPTS

- Structural welding is a process by which the parts that are to be connected are heated and fused, with supplementary molten metal at the joint.
- A relatively small depth of material will become molten, and upon cooling, the structural steel and weld metal will act as one continuous part where they are joined.







- The additional metal is deposited from a special electrode, which is part of the electric circuit that includes the connected part.
  - In the shielded metal arc welding (SMAW) process, *current* arcs across a gap between the electrode and the base metal, heating the connected parts and depositing part of the electrode into the molten base metal.
  - A special coating on the electrode vaporizes and forms a protective gaseous shield, preventing the molten weld metal from oxidizing before it solidifies.
  - The electrode is moved across the joint, and a weld bead is deposited, its size depending on the rate of travel of the electrode.
  - As the weld cools, impurities rise to the surface, forming a coating called *slag* that must be removed before the member is painted or another pass is made with the electrode.
  - Shielded metal arc welding is usually done manually and is the process universally used for field welds.
- For shop welding, an automatic or semi automatic process is usually used. Foremost among these is the submerged arc welding (SAW),
- In this process, the end of the electrode and the arc are submerged in a granular flux that melts and forms a gaseous shield. There is more penetration into the base metal than with shielded metal arc welding, and higher strength results.
- Other commonly used processes for shop welding are *gas shielded metal arc*, *flux cored arc*, and *electro-slag welding*.
- Quality control of welded connections is particularly difficult, because defects below the surface, or even minor flaws at the surface, will escape visual detection. Welders must be

properly certified, and for critical work, special inspection techniques such as radiography or ultrasonic testing must be used.

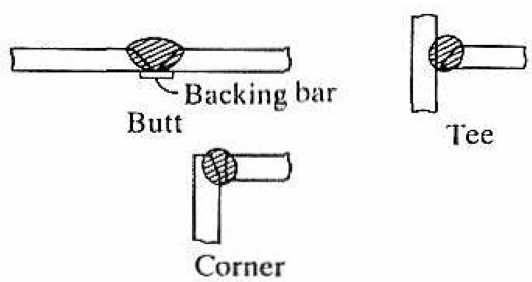
- The two most common types of welds are the fillet weld and the groove weld. Fillet weld examples: lap joint – fillet welds placed in the corner formed by two plates

  Tee joint – fillet welds placed at the intersection of two plates.

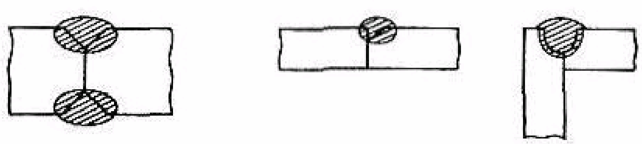
- Groove welds – deposited in a gap or groove between two parts to be connected

  e.g., butt, tee, and corner joints with beveled (prepared) edges

- Partial penetration groove welds can be made from one or both sides with or without edge preparation.



(a) Complete Penetration Groove Welds

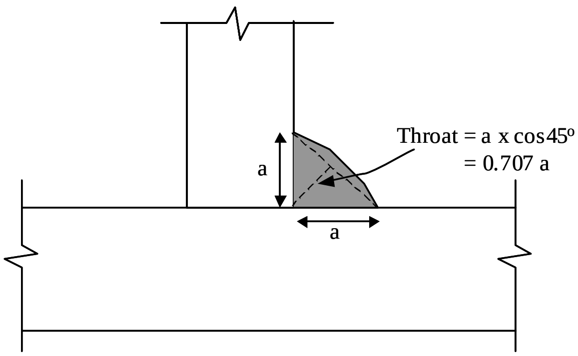


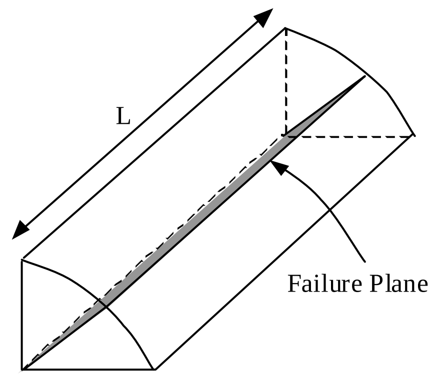
(b) Partial Penetration Groove Welds

6.2 Design of Welded Connections

- Fillet welds are most common and used in all structures.
- Weld sizes are specified in 1/16 in. increments
- A fillet weld can be loaded in any direction in shear, compression, or tension. However, it always **<u>fails in *shear*</u>**.

- The shear failure of the fillet weld occurs along a plane through the throat of the weld, as shown in the Figure below.





- Shear stress in fillet weld of length L subjected to load P = $f_v = \dfrac{P}{0.707\, a\, L_w}$

- If the ultimate shear strength of the weld = f_w

  $R_n = f_w \times 0.707 \times a \times L_w$

  $\phi R_n = 0.75 \times f_w \times 0.707 \times a \times L_w$ i.e., ϕ factor = 0.75

- f_w = shear strength of the weld metal is a function of the electrode used in the SMAW process.

  - The tensile strength of the weld electrode can be 60, 70, 80, 90, 100, 110, or 120 ksi.
  - The corresponding electrodes are specified using the nomenclature E60XX, E70XX, E80XX, and so on. This is the standard terminology for weld electrodes.

- The strength of the electrode should match the strength of the *base metal*.

  - If yield stress (σ_y) of the base metal is ≤ 60 - 65 ksi, use E70XX electrode.
  - If yield stress (σ_y) of the base metal is ≥ 60 - 65 ksi, use E80XX electrode.

- E70XX is the most popular electrode used for fillet welds made by the SMAW method.

- **Table J2.5** in the AISC Specifications gives the weld design strength

  $f_w = 0.60\, F_{EXX}$

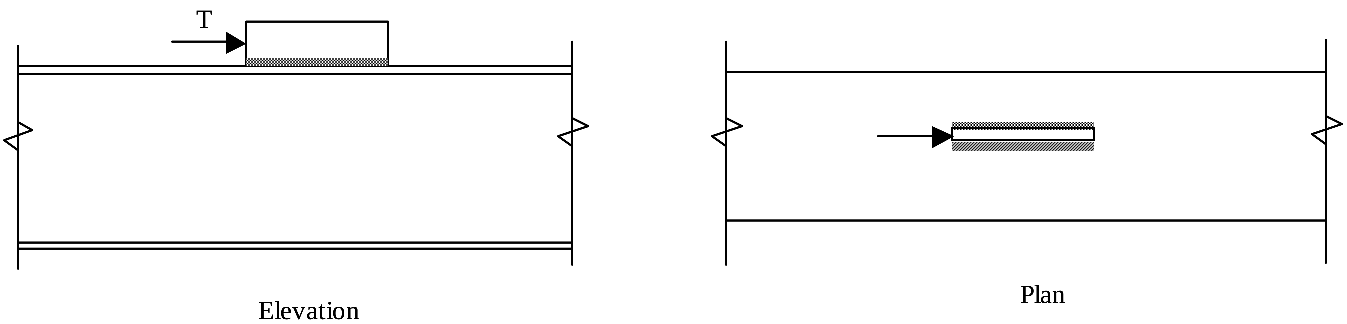
For E70XX, $\phi f_w = 0.75 \times 0.60 \times 70 = 31.5$ ksi

- Additionally, the shear strength of the base metal must also be considered:

  $\phi R_n = 0.9 \times 0.6\ F_y$ x area of base metal subjected to shear

  where, F_y is the yield strength of the base metal.

- For example:



Strength of weld in shear
$= 0.75 \times 0.707 \times a \times L_w \times f_w$

Strength of base metal
$= 0.9 \times 0.6 \times F_y \times t \times L_w$

Smaller governs the strength of the weld

- Always check weld metal and base metal strength. Smaller value governs. In most cases, the weld metal strength will govern.
- In weld design problems it is advantageous to work with strength per unit length of the weld or base metal.

6.2.1 Limitations on weld dimensions (See AISC Spec. **J2.2b** on page **16.1-54** of manual)

- **Minimum size (a_{min})**
  - function of the thickness of the thickest connected plate
  - given in Table J2.4 of the AISC specifications
- **Maximum size (a_{max})**

- function of the thickness of the thinnest connected plate:

- for plates with thickness ≤ 0.25 in., a_{max} = 0.25 in.

- for plates with thickness ≥ 0.25 in., a_{max} = t - 1/16 in.

- **Minimum length (L_w)**

- length (L_w) ≥ 4 a otherwise, $a_{eff} = L_w / 4$

- Read **J2.2 b**

- Intermittent fillet welds: $L_{w\text{-}min}$ = 4 a <u>and</u> 1.5 in.

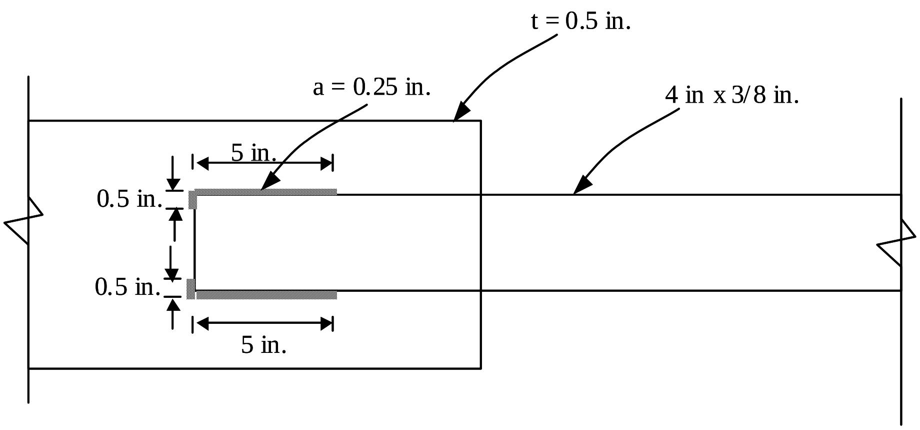
- **Maximum effective length - read AISC J2.2b**

- If weld length L_w < 100 a, then effective weld length ($L_{w\text{-}eff}$) = L_w
- If L_w < 300 a, then effective weld length ($L_{w\text{-}eff}$) = L_w (1.2 – 0.002 L_w/a)
- If L_w > 300 a, the effective weld length ($L_{w\text{-}eff}$) = 0.6 L_w

- **Weld Terminations - read AISC J2.2b**

- Lap joint – fillet welds terminate at a distance > a from edge.
- Weld returns around corners must be > 2 a

Example 6.1. Determine the design strength of the tension member and connection system shown below. The tension member is a 4 in. x 3/8 in. thick rectangular bar. It is welded to a 1/2 in. thick gusset plate using E70XX electrode. Consider the yielding and fracture of the tension member. Consider the shear strength of the weld metal and the surrounding base metal.



<u>Solution</u>

Step I. <u>Check for the limitations on the weld geometry</u>

- t_{min} = 3/8 in. (member)

  t_{max} = 0.5 in. (gusset)

  Therefore, a_{min} = 3/16 in. - AISC **Table J2.4**

  a_{max} = 3/8 - 1/16 = 5/16 in. - AISC **J2.2b**

  Fillet weld size = a = 1/4 in. - *Therefore, OK!*

- L_{w-min} = 1.0 in. - OK.
  - L_{w-min} for each length of the weld = 4.0 in. (*transverse distance between welds*, see **J2.2b**)
  - Given length = 5.0 in., which is > L_{min}. Therefore, *OK*!

- Length/weld size = 5/0.25 = 20 - *Therefore, maximum effective length* **J2.2 b** *satisfied.*

- End returns at the edge corner size - minimum = 2 a = 0.5 in. -*Therefore, OK!*

Step II. Design strength of the weld

- Weld strength = ϕ x 0.707 x a x 0.60 x F_{EXX} x L_w

  = 0.75 x 0.707 x 0.25 x 0.60 x 70 x 10 = 55.67 kips

- Base Metal strength = ϕ x 0.6 x F_y x L_w x t

  = 0.9 x 0.6 x 50 x 10 x 3/8 = 101.25 kips

Step III. Tension strength of the member

- ϕR_n = 0.9 x 50 x 4 x 3/8 = 67.5 kips - tension yield

- ϕR_n = 0.75 x A_e x 65 - tension fracture

  A_e = U A

  A = A_g = 4 x 3/8 = 1.5 in^2 - See Spec. B3

  U = 0.75 , since connection length (L_{conn}) < 1.5 w - See Spec. B3

  Therefore, ϕR_n = 54.8 kips

The design strength of the member-connection system = 54.8 kips. Tension fracture of the member governs. The end returns at the corners were not included in the calculations.

Example 6.2 Design a double angle tension member and connection system to carry a factored load of 250 kips.

Solution

Step I. Assume material properties

- Assume 36 ksi steel for designing the member and the gusset plates.
- Assume E70XX electrode for the fillet welds.

Step II. Design the tension member

- From Table 3-7 on page 3-32 of the AISC manual, select 2*L* 5 x 3½ x 1/2 made from 36 ksi steel with yield strength = 259 kips and fracture strength = 261 kips.

Step III. Design the welded connection

- a_{min} = 3/16 in. - **Table J2.4**

  a_{max} = 1/2 - 1/16 in. = 7/16 in. - **J2.2b**

  Design, a = 3/8 in. = 0.375 in.

- Shear strength of weld metal = $\phi R_n = 0.75 \times 0.60 \times F_{EXX} \times 0.707 \times a \times L_w$

  $= 8.35\ L_w$ kips

- Strength of the base metal in shear = $\phi R_n = 0.9 \times 0.6 \times F_y \times t \times L_w$

  $= 9.72\ L_w$ kips

- Shear strength of weld metal governs, $\phi R_n = 8.35\ L_w$ kips
- $\phi R_n > 250$ kips

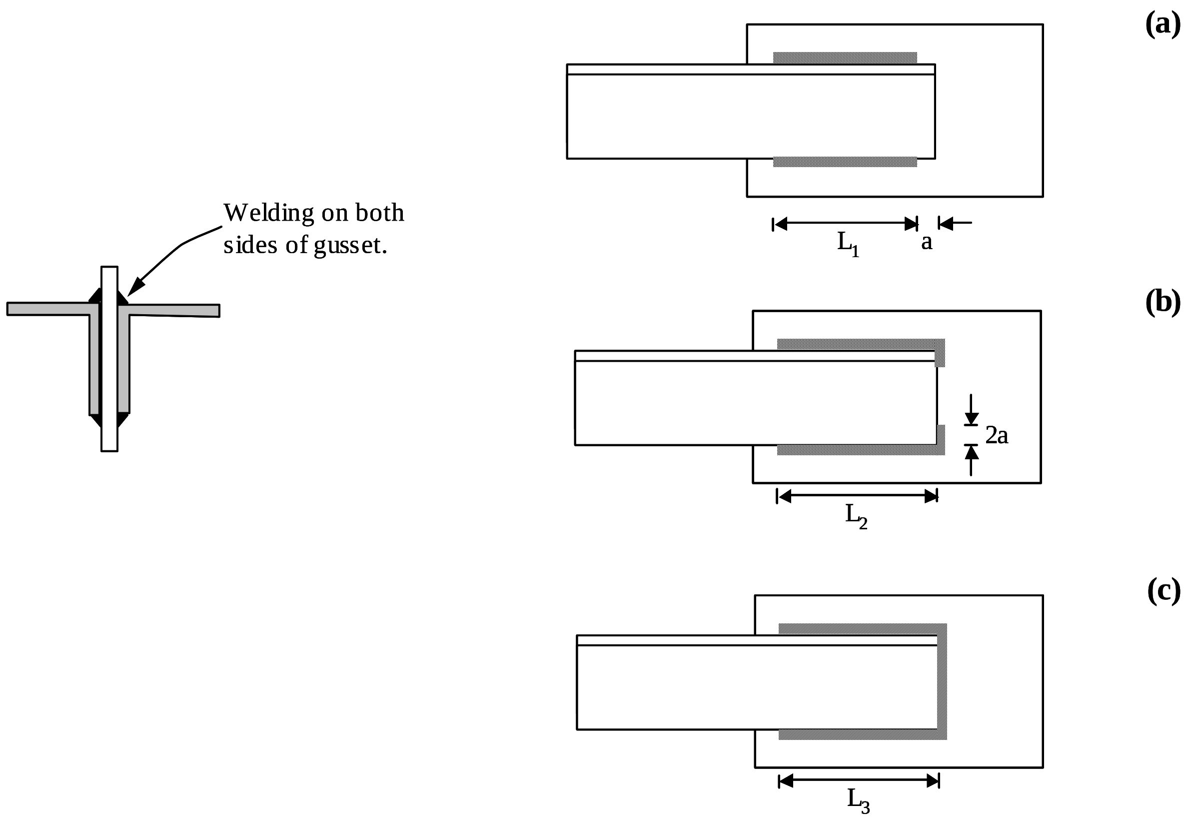
  $\therefore\ 8.35\ L_w > 250$ kips

  $\therefore\ L_w > 29.94$ in.

  Design, length of 1/2 in. E70XX fillet weld = 30.0 in.

- *Shear strength of fillet weld = 250.5 kips*

Step IV. Layout of Connection



- Length of weld required = 30 in.

  Since there are two angles to be welded to the gusset plate, assume that total weld length for each angle will be 15.0 in.

- As shown in the Figure above, 15 in. of 1/2 in. E70XX fillet weld can be placed in three ways (a), (b), and (c).

  - For option (a), the AISC Spec. **J2.2b** requires that the fillet weld terminate at a distance greater than the size (1/2 in.) of the weld. For this option, L_1 will be equal to 7.5 in.

  - For option (b), the AISC Spec. **J2.2b** requires that the fillet weld be returned continuously around the corner for a distance of at least 2 a (1 in.). For this option, L_2 can be either 6.5 in. or 7.5 in. However, the value of 7.5 in. is preferred.

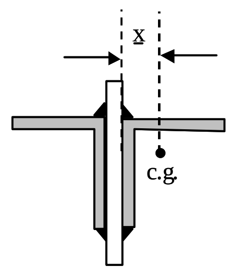
  - For option (c), L_3 will be equal to 5.75 in.

Step V. Fracture strength of the member

- $A_e = U\, A_g$

For the double angle section, use the value of x from Table 1-7 on page 1-37 of manual.

Option	$U = 1 - \frac{\bar{x}}{L}$
(a)	1-0.901/7.5 = 0.88 ≤ 0.9
(b)	1-0.901/6.5 =0.86 ≤ 0.9
(c)	1-0.901/5.75 = 0.84 ≤ 0.9



Assume case (a). Therefore, U =0.88

$\phi\, R_n = 0.75 \times 0.88 \times 8.00 \times 58 = 306.24$ kips > 250 kips - fracture limit state is *ok!*

Step VI. Design the gusset plate

$\phi R_n > T_u$ - tension yielding limit state

Therefore, $0.9 \times A_g \times 36 > 250$ kips

$A_g > 7.71\ in^2$

$\phi R_n > T_u$ - tension fracture limit state

Therefore, $0.75 \times A_n \times F_u > 250$ kips

$A_n \leq 0.85\ A_g$ - Spec. **J5**

$A_n > 5.747\ in^2$ Therefore, $A_g > 6.76\ in^2$

Design gusset plate thickness

= 1.0 in. and width = 8.0 in.

(a)