University of Pretoria

School of Engineering

Department of Civil and Biosystems Engineering

Reinforced Concrete Structures

Formulas and Tables for SABS 0100:1992

John M. Robberts

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This formula book is intended as an aid to students during tests and exams. It therefore contains a summary of only the most important design equations and does not replace the design code of practice SABS 0100 to which designers should refer.

Table of Contents

1 Material Properties

1.1 Concrete

Modulus of elasticity

$$E_{c,t} = E_{c,28}\left(0.4 + 0.6\frac{f_{cu,t}}{f_{cu,28}}\right) \quad (1\text{-}1)$$

Poisson's ratio $\nu = 0.2$ (1-2)

Coefficients of thermal expansion

$$\alpha_{th} = 10 \times 10^{-6}\ ^{\circ}\mathrm{C}^{-1} \quad (1\text{-}3)$$

Unit weight $\gamma_c = 24\ \mathrm{kN/m^3}$ (1-4)

Shear modulus $G = \dfrac{E_c}{2(1+\nu)} \approx 0.4E_c$ (1-5)

Total long-term concrete strain

$$\varepsilon_{c,tot}(t) = \varepsilon_{el}(t) + \varepsilon_{cr}(t) + \varepsilon_{sh}(t) + \varepsilon_{th}(t) \quad (1\text{-}6)$$

Table 1: Secant modulus of elasticity at 28 days $E_{c,28}$ (GPa)

Characteristic cube strength $f_{cu,28}$ (MPa)	**Average**	**Typical range**
20	25	21 - 29
25	26	22 - 30
30	28	23 - 33
40	31	26 - 36
50	34	28 - 40
60	36	30 - 42

Shrinkage strain $\varepsilon_{sh}(t) = \varepsilon_{sh,\infty}\left(1 - e^{-\alpha t}\right)$ where $\varepsilon_{sh,\infty}$ ranges from 0.10 to 0.30 × 10^{-3} (1-7)

Creep strain $\varepsilon_{cr}(t) = f_c\, C(t) = \dfrac{f_c}{E_c}\phi(t)$ (1-8)

Creep coefficient $\phi(t) = \phi_\infty\left(1 - e^{-\alpha t}\right)$ where ϕ_∞ ranges from 1.5 to 3.5. (1-9)

Elastic and creep strain $\varepsilon_{el} + \varepsilon_{cr}(t) = \dfrac{f_c}{E_{eff}(t)}$ where $E_{eff}(t) = \dfrac{E_c}{1+\phi(t)}$ (1-10)

1.2 Reinforcement

Table 2: Reinforcement types.

Reinforcement type	**Symbol**	**Minimum characteristic yield strength f_y (MPa)**
Hot rolled mild steel	R	250
Hot rolled high-yield steel	Y	450
Cold worked high-yield steel	Y	450
Welded wire fabric	FS or FD	485

Modulus of elasticity: $E_s = 200$ GPa

For reinforcement areas see Tables 31 and 30.

2 Limit States Design

Table 3: Partial factors of safety for materials γ_m.

Limit state		Concrete	Steel
Ultimate	Flexure, axial load	1.5	1.15
	Shear	1.4	1.15
	Bond	1.4	
Serviceability:		1.0	1.0

Design load effects ≤ Design load effects $\gamma_f Q_n \leq f_k / \gamma_m$ (2-1)

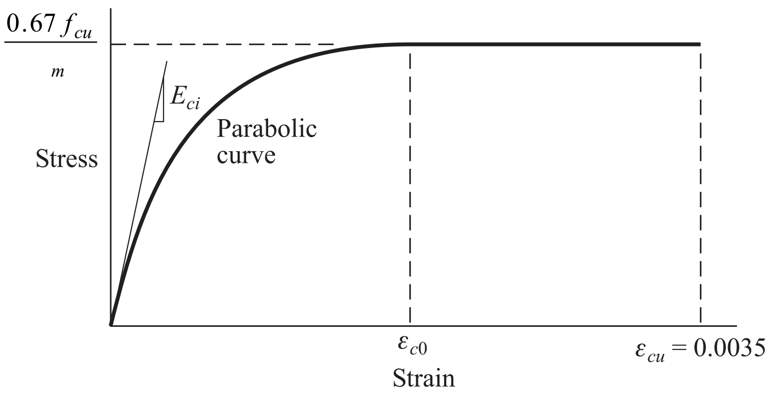
Characteristic strength $f_k = f_m - 1.64\sigma$ (2-2)

Table 4: Typical* partial factors of safety for loads γ_f.

Load combination	Ultimate limit state			Serviceability limit state		
	Self-weight load	**Imposed load**	**Wind load**	**Self-weight load**	**Imposed load**	**Wind load**
Self-weight load	1.5 (0.9)	–	–	1.1 (1.0)	–	–
Self-weight load + live load	1.2 (0.9)	1.6 (0)	–	1.1 (1.0)	1.0 (0)	–
Self-weight load + live load + wind load	1.2 (0.9)	0.5 (0)	1.3 (1.3)	1.1 (1.0)	0.3 (0)	0.6 (0.6)

* See SABS 0160 (1989) for a complete discussion on loads and their combinations.

3 Analysis and Design for Flexure



$E_{ci}\quad 5.5\sqrt{\frac{f_{cu}}{m}}$ GPa

$_{c0}\quad 2.4\quad 10^{4}\sqrt{\frac{f_{cu}}{m}}$

f_{cu} in MPa

Figure 1: Parabolic-rectangular stress-strain relationship for concrete in flexure.

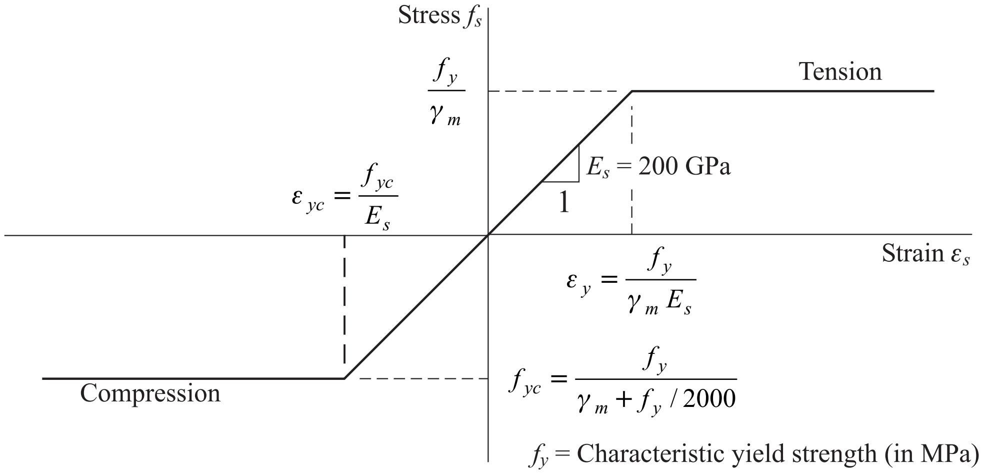


Figure 2: Stress-strain relationship for reinforcement.

Table 5: Yield stress and strain for reinforcement.

Reinforcement type	**Symbol**	**Tension**		**Compression**	
		Yield strength f_y	**Yield strain ε_y ($\times 10^{-3}$)**	**Yield strength f_{yc}**	**Yield strain ε_{yc} ($\times 10^{-3}$)**
Mild steel	R	250	1.087	196.1	0.980
High yield	Y	450	1.957	327.3	1.636
Welded wire fabric	FS or FD	485	2.109	348.3	1.741
The values in this table includes γ_m = 1.15.					

3.1 Relationship between strains and neutral axis depth

Strain in tension reinforcement

$$\varepsilon_{st} = \varepsilon_c \left(\frac{d - x}{x} \right) \tag{3-1}$$

Strain in compression reinforcement

$$\varepsilon_{sc} = \varepsilon_c \left(\frac{x - d'}{x} \right) \tag{3-2}$$

Neutral axis/reinforcement depth

$$\frac{x}{d} = \frac{\varepsilon_c}{\varepsilon_{st} + \varepsilon_c} \tag{3-3}$$

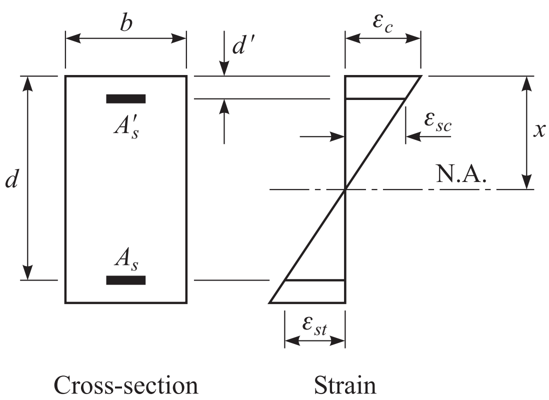


Figure 3: Relationship between strains and neutral axis depth.

3.2 Singly reinforced rectangular sections

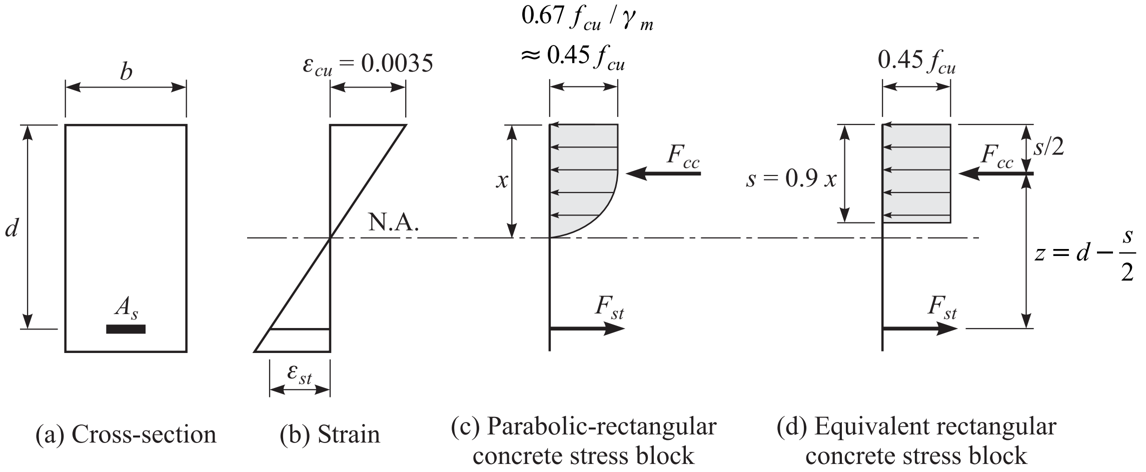


Figure 4: Equivalent rectangular stress block for singly reinforced rectangular sections.

Neutral axis depth

$$x \le (\beta_b - 0.4)\,d \le 0.5 \qquad (3\text{-}4)$$

where

$$\beta_b = \left(\frac{\text{Moment at the section following redistribution}}{\text{Moment at the section before redistribution}} \right) \qquad (3\text{-}5)$$

with $\beta_b \le 1$ (3-6)
> 0.75 under normal conditions
> 0.8 if the cross-section varies along the member
> 0.9 for structures exceeding 4 storeys

Table 6: Moment redistribution and limits on neutral axis depth.

Redistrib. (%)	β_b	x/d	z/d	K'
≤ 10	0.90	0.50	0.775	0.156
15	0.85	0.45	0.797	0.144
20	0.80	0.40	0.820	0.132
25	0.75	0.35	0.842	0.119

For tension reinforcement only $\quad K = \dfrac{M}{f_{cu}\, b\, d^2} \le K'$

(3-7)

where

$$K' = \begin{cases} 0.156 & \text{for } \beta_b \ge 0.9 \\ 0.402\,(\beta_b - 0.4) - 0.18(\beta_b - 0.4)^2 & \text{for } \beta_b < 0.9 \end{cases} \qquad (3\text{-}8)$$

Internal lever-arm

$$z = d\left[0.5 + \sqrt{\left(0.25 - \frac{K}{0.9}\right)}\right] \le 0.95\,d \qquad (3\text{-}9)$$

Required area of reinforcement

$$A_s = \frac{M}{0.87 f_y z} \qquad (3\text{-}10)$$

3.3 Doubly reinforced rectangular sections

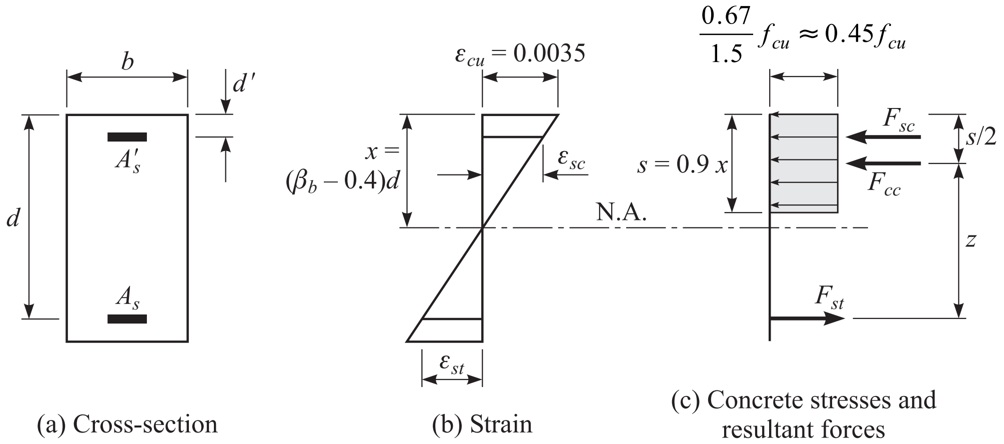


Figure 5: Doubly reinforced rectangular concrete section.

Compression reinforcement must be provided if $K > K'$

Required area of reinforcement $$A'_s = \frac{(K - K')\, b d^2\, f_{cu}}{f_{yc}\,(d - d')} \qquad (3\text{-}11)$$

$$A_s = \frac{K'\, f_{cu}\, b d^2}{0.87\, f_y\, z} + \frac{f_{yc}}{0.87\, f_y} A'_s \qquad (3\text{-}12)$$

where $$z = d\left[0.5 + \sqrt{0.25 - \frac{K'}{0.9}}\right] \quad \text{and} \quad x = \frac{2}{0.9}(d - z) \qquad (3\text{-}13, 14)$$

Table 7: Conditions whereby reinforcement yield.

Yield strength f_y (MPa)	Tension reinforcement yields when $x/d \le$	Compression reinforcement yields when				
		$d'/x \le$	$d'/d \le$			
			$\beta_b = 0.90$	$\beta_b = 0.85$	$\beta_b = 0.80$	$\beta_b = 0.75$
250	0.7629	0.7199	0.3599	0.3239	0.2880	0.2520
450	0.6414	0.5325	0.2662	0.2396	0.2130	0.1864
485	0.6239	0.5024	0.2512	0.2261	0.2010	0.1759

Compression reinforcement yields when $\dfrac{d'}{d} \le \left(1 - \dfrac{f_{yc}}{E_s\, \varepsilon_{cu}}\right)(\beta_b - 0.4)$

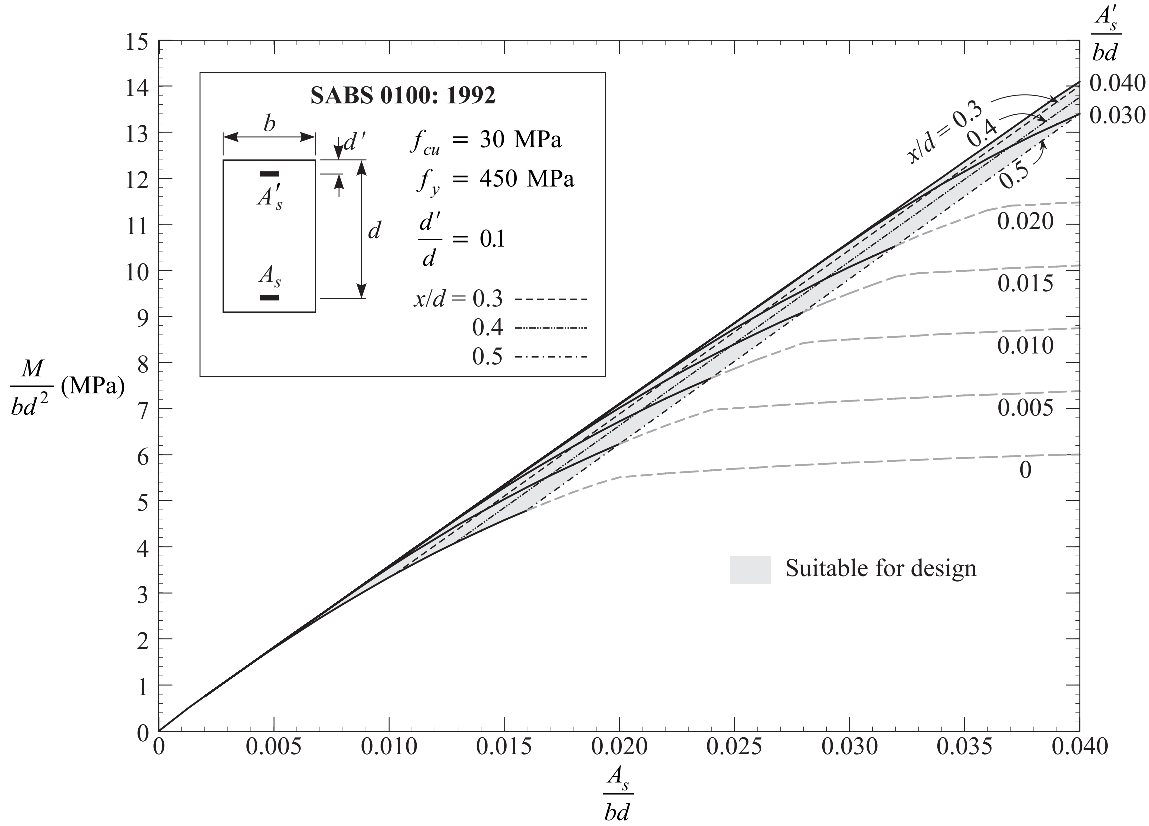


Figure 6: Design chart for flexure.

3.4 Flanged beams

Neutral axis within the web ($x > h_f$): Solve s_w from

$$M = 0.45\, f_{cu}\, b_f\, h_f \left(d - \frac{h_f}{2}\right) + 0.45\, f_{cu}\, b_w\, s_w \left(d - h_f - \frac{s_w}{2}\right) \tag{3-15}$$

$$A_s = \frac{0.45\, f_{cu}\, b_f\, h_f + 0.45\, f_{cu}\, b_w\, s_w}{0.87\, f_y} \tag{3-16}$$

Simplified design for $x > h_f$ with $x = d/2$:

$$A_s = \frac{M + 0.1\, f_{cu}\, b_w\, d\left(0.45\,d - h_f\right)}{0.87\, f_y \left(d - 0.5\, h_f\right)} \tag{3-17}$$

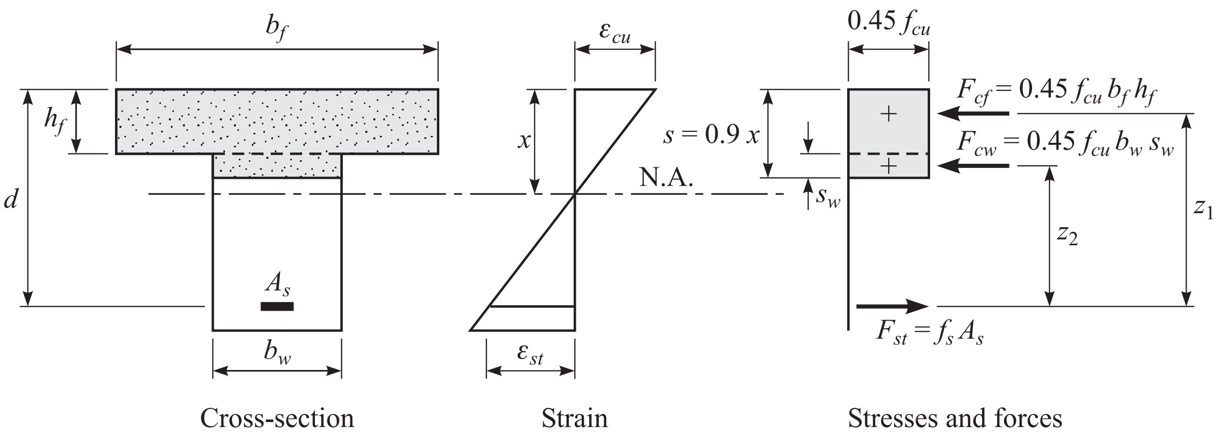


Figure 7: T-section with neutral axis within the web.

3.5 Elastic analysis and design

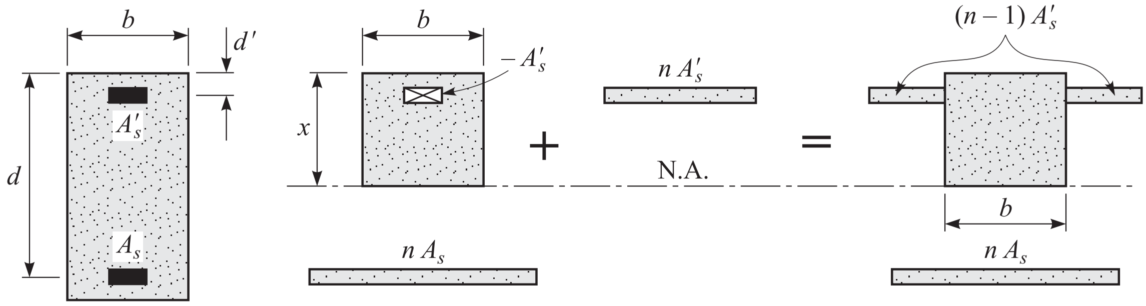
Cracked rectangular section

Find x from $$b x \frac{x}{2} + (n-1)\, A_s' (x - d') = n\, A_s (d - x) \quad \text{where} \quad n = \frac{E_s}{E_c} \tag{3-18}$$

Cracked transformed second moment of area

$$I_{cr} = \frac{1}{3} b x^3 + (n-1)\, A_s' (x - d')^2 + n\, A_s (d - x)^2 \tag{3-19}$$

Stresses $$f_{cc} = \frac{M\,x}{I_{cr}}, \quad f_{st} = n\, f_{ec} = n\left[\frac{M\,(d-x)}{I_{cr}}\right], \quad f_{sc} = n\left[\frac{M\,(x-d')}{I_{cr}}\right] \tag{3-20}$$



(a) Cross-section (b) Transformed section

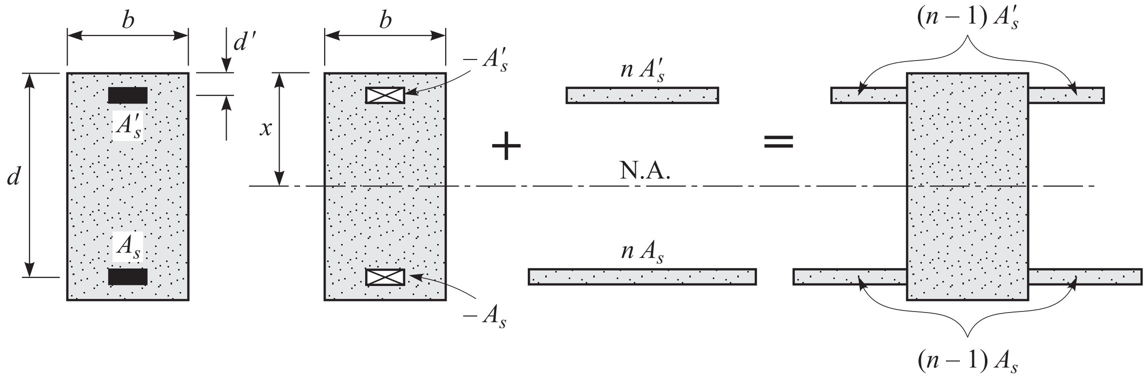
Figure 9: Cracked transformed rectangular section with compression reinforcement.

Uncracked section

Find x from $$b \frac{x^2}{2} + (n-1)\, A_s' (x - d') = b \frac{(h-x)^2}{2} + (n-1)\, A_s (d - x) \tag{3-21}$$

Uncracked transformed second moment of area

$$I_{co} = \frac{1}{3} b x^3 + \frac{1}{3} b (h-x)^3 + (n-1)\, A_s' (x - d')^2 + (n-1)\, A_s (d - x)^2 \tag{3-22}$$



(a) Cross-section (b) Transformed section

Figure 8: Uncracked transformed rectangular section with compression reinforcement.

4 Design of Beams for Shear

Step 1: Ultimate design shear stress $$v = \frac{V}{b_v\, d} \tag{4-1}$$

where b_v = width of the beam (average width of the web below the flange for a T-section)

Step 2: Check that $v < v_u$ were $$v_u = \text{lesser of} \begin{cases} 0.75\sqrt{f_{cu}} \\ 4.75\,\text{MPa} \end{cases} \tag{4-2}$$

Step 3: Shear capacity of the beam without shear reinforcement

$$v_c = \frac{0.75}{\gamma_m}\left(\frac{f_{cu}}{25}\right)^{1/3}\left(\frac{100\,A_s}{b_v\, d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \tag{4-3}$$

where $\gamma_m = 1.4$ $\quad f_{cu} \le 40$ MPa $\quad \dfrac{100\,A_s}{b_v\, d} \le 3$

A_s = area of properly anchored tension reinforcement (distance d beyond the section under consideration)

Table 8: Shear capacity v_c (in MPa) for beams without shear reinforcement for f_{cu} = 30 MPa.

$\frac{100A_s}{b_v d}$	Effective depth d (mm)									
	125	150	175	200	225	250	300	400	500	800
≤ 0.15	0.4046	0.3865	0.3719	0.3597	0.3493	0.3402	0.325	0.3025	0.2861	0.2544
0.25	0.4797	0.4583	0.441	0.4265	0.4141	0.4033	0.3854	0.3586	0.3392	0.3016
0.50	0.6043	0.5774	0.5556	0.5373	0.5217	0.5082	0.4855	0.4518	0.4273	0.3799
0.75	0.6918	0.661	0.6360	0.6151	0.5972	0.5817	0.5558	0.5172	0.4892	0.4349
1.00	0.7614	0.7275	0.7000	0.677	0.6573	0.6403	0.6117	0.5693	0.5384	0.4787
1.50	0.8716	0.8328	0.8013	0.775	0.7525	0.7329	0.7003	0.6517	0.6163	0.548
2.00	0.9593	0.9166	0.8819	0.853	0.8282	0.8067	0.7707	0.7172	0.6783	0.6031
≥ 3.00	1.0981	1.0492	1.0095	0.9764	0.9481	0.9234	0.8823	0.821	0.7765	0.6904

Step 4:

Nominal shear reinforcement: $$\left(\frac{A_{sv}}{s_v}\right) > \begin{cases} 0.0020\,b & \text{for } f_{yv} = 250\,\text{MPa} \\ 0.0012\,b & \text{for } f_{yv} = 450\,\text{MPa} \end{cases} \tag{4-4}$$

Step 5: If $v > v_c$, shear reinforcement must be provided

For bent-up bars: $$V_s = A_{sb}\left(0.87 f_{yv}\right)\left[\cos\alpha + \sin\alpha\, \cot\beta\right]\left(\frac{d-d'}{s_b}\right) \tag{4-5}$$

or $$\frac{A_{sb}}{s_b} \ge \frac{\left(v - v_c\right) b_v}{\left(0.87 f_{yv}\right)\left[\cos\alpha + \sin\alpha\, \cot\beta\right]} \tag{4-6}$$

where A_{sb} = area of a bent-up bar, s_b = spacing of bent-up bars, α = angle between the horizontal and the bar and β = angle between compressive struts in the concrete and the horizontal.

For vertical links: $$\frac{A_{sv}}{s_v} \geq \frac{(v - v_c)\, b}{0.87\, f_{yv}} \quad \text{with } f_{yv} \leq 450\ \text{MPa.} \qquad (4\text{-}7)$$

Table 9: A_{sv}/s_v (mm²/mm) for links with two legs

Diameter (mm)	Link spacing (mm)										
	75	100	125	150	200	250	300	350	400	450	500
8	1.340	1.005	0.804	0.670	0.503	0.402	0.335	0.287	0.251	0.223	0.201
10	2.094	1.571	1.257	1.047	0.785	0.628	0.524	0.449	0.393	0.349	0.314
12	3.016	2.262	1.810	1.508	1.131	0.905	0.754	0.646	0.565	0.503	0.452
16	5.362	4.021	3.217	2.681	2.011	1.608	1.340	1.149	1.005	0.894	0.804

Step 6: $$\text{Maximum spacing} = \begin{cases} \leq 0.75\, d & \text{for links} \\ \leq 1.5\, d & \text{for bent-up bars} \end{cases} \qquad (4\text{-}8)$$

Further comments:

- Within a distance 2d from a support, or concentrated load, the shear resistance v_c may be increased as follows

$$v_c\left(\frac{2\,d}{a_v}\right) \leq \text{ lesser of } \begin{cases} 0.75\sqrt{f_{cu}} \\ 4.75\ \text{MPa} \end{cases} \qquad (4\text{-}9)$$

- For beams carrying mainly uniformly distributed loads, or where the principal load is applied further than 2d from the face of the support, a critical section at a distance d, from the face of the support is considered.
- For slabs thinner than 200 mm the resistance of the shear reinforcement should be reduced by 10 % for every 10 mm reduction of slab thickness below 200 mm.

5 Design for Torsion

Step 1: Find A_s and A_{sv} to resist bending and shear.

Step 2: Find the torsional shear stress from $$v_t = \frac{2T}{h_{min}^2\left(h_{max} - \frac{h_{min}}{3}\right)} \qquad (5\text{-}1)$$

where h_{min} is the smaller and h_{max} is the larger section dimension.

Step 3: Divide T- L- and I-sections into components that maximises $\sum\left(h_{min}^3\, h_{max}\right)$. Consider each component individually, subjected to a moment

$$T_i = T \frac{\left(h_{min}^3\, h_{max}\right)_i}{\sum \left(h_{min}^3\, h_{max}\right)_i} \tag{5-2}$$

Step 4: Check that $$v_t + v \le v_{tu} \tag{5-3}$$

where v is the shear stress equal to $V/(bd)$.

Also check for small sections that $$v_t \le v_{tu}\left(\frac{y_1}{550\,\text{mm}}\right) \tag{5-4}$$

Sep 5: Provide torsional reinforcement if $v_t > v_{t,min}$ with $v_{t,min}$ from Table 10. For combined shear and torsion refer to Table 11.

Table 10: Minimum and maximum stresses for torsion (MPa).

	f_{cu} (MPa)			
	20	25	30	≥ 40
$v_{t,min}$	0.27	0.30	0.33	0.36
v_{tu}	3.18	3.56	4.00	$4.50 < v_{tu} < 4.75$
where $v_{tu} = 0.71\sqrt{f_{cu}} \le 4.75$ MPa and $v_{t,min} = 0.06\sqrt{f_{cu}} \le 0.36$ MPa These values includes $\gamma_m = 1.4$				

Table 11: Reinforcement for combined shear and torsion.

	$v_t \le v_{t,min}$	$v_t > v_{t,min}$
$v \le (v_c + 0.4)$	Minimum shear reinforcement; no torsion reinforcement	Designed torsion reinforcement but not less than minimum shear reinforcement
$v > (v_c + 0.4)$	Designed shear reinforcement; no torsion reinforcement	Designed shear and torsion rein-forcement

Step 6: Designed torsion reinforcement (additional to that required for shear in step 1)

$$\frac{A_{sv}}{s_v} = \frac{T}{0.8 x_1\, y_1 (0.87 f_{yv})} \tag{5-5}$$

$$A_s \ge \frac{A_{sv}}{s_v}\left(\frac{f_{yv}}{f_y}\right)\left(x_1 + y_1\right) \tag{5-6}$$

Step 7: Detailing requirements:

- Only use closed links for torsion.
- Maximum spacing for links is the lesser of: x_1, $y_1/2$ or 200 mm.

- Longitudinal torsion reinforcement must be distributed evenly around the inside perimeter of the links so that the maximum clear distance between bars is less than 300 mm.
- Each corner of a link should contain at least one longitudinal bar.
- Torsion reinforcement may be included at levels of existing flexural reinforcement by increasing the diameters of the flexural reinforcement appropriately.
- Torsion reinforcement should extend for at least a distance equal to the largest section dimension beyond the point where it is theoretically required.
- For T-, L- and I-sections the reinforcement should be detailed so that they interlock and tie the component rectangles together. If $v_t < v_{t,min}$ for a smaller component rectangle, torsion reinforcement may be omitted for that component.

6 Bond and Anchorage

Anchorage bond length $$L = \frac{f_s}{4 f_{bu}} \phi \qquad (6\text{-}1)$$

Bearing stress inside a bend $$\frac{F_{bt}}{r\phi} \leq \frac{2 f_{cu}}{1 + 2(\phi / a_b)} \qquad (6\text{-}2)$$

Lapping of bars:

- Minimum lap length must be the greater of 15 ϕ or 300 mm for bars and 250 mm for fabric.
- Lap lengths for bars of different diameters can be based on the smaller diameter.
- Compression laps must be 25% greater than design anchorage length in compression.
- When both bars in a lap are 25 mm or greater in size, and the cover is less than 1.5 times the smaller bar size, then transverse links of at least 1/4 of the smaller bar size should be provided at a maximum spacing of 200 mm.
- If bars are placed in a bundle, only one bar at a time may be lapped. The maximum number of bars in a bundle, including laps, should not be more than 4.

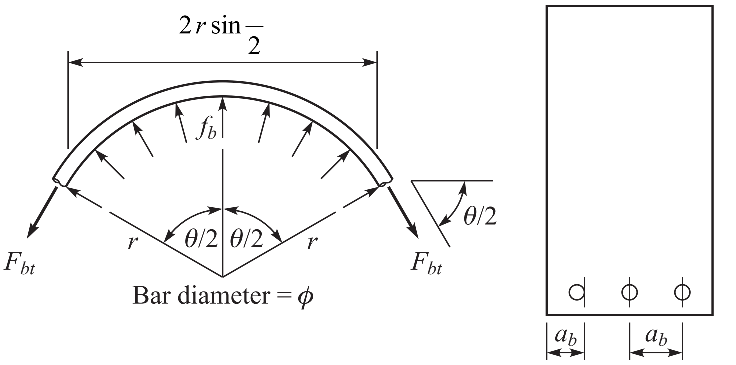
Table 12: Design ultimate bond stress f_{bu} in MPa (SABS 0100).

Bar type	**Concrete grade**			
	20	**25**	**30**	**≥ 40**
Plain bar in tension	1.2	1.4	1.5	1.9
Plain bar in compression	1.5	1.7	1.9	2.3
Deformed bar in tension	2.2	2.5	2.9	3.4
Deformed bar in compression	2.7	3.1	3.5	4.2
Reduce these values by 30% for deformed top bars and by 50% for plain top bars in elements where the depth exceeds 300 mm.				

Table 13: Ultimate anchorage bond lengths as multiples of bar sizes.

Bar type	Concrete grade			
	20	**25**	**30**	**≥ 40**
Plain bar[1] in tension	46	39	37	29
Plain bar[1] in compression	37	32	29	24
Deformed bar[2] in tension	45	40	34	29
Deformed bar[2] in compression	37	32	28	24

[1] Mild steel f_y = 250 MPa
[2] High yield steel f_y = 450 MPa
Reduce these values by 30% for deformed top bars and by 50% for plain top bars in elements where the depth exceeds 300 mm.



(a) Equilibrium of forces (b) Definition of a_b

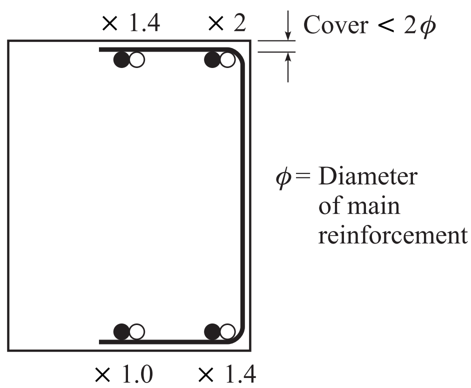


Figure 10: Bearing stress inside a bend.

Figure 11: Increasing lap lengths

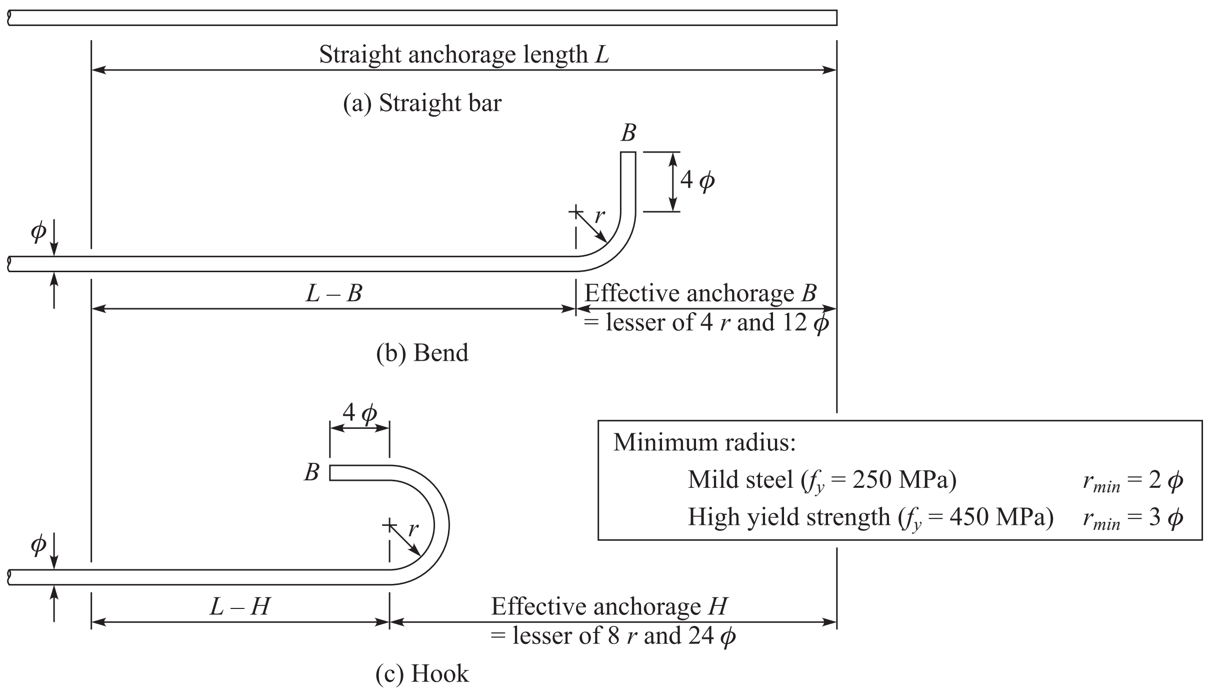


Figure 12: Equivalent anchorage of a hook and a bend.

7 Design for Serviceability

7.1 Cover to concrete

Table 14: Minimum cover (in mm) for various exposure conditions.

Concrete	Conditions of exposure				
	Mild	Moderate	Severe	Very severe	Extreme
Normal density concrete[1]	20	30	40	50	60
Low-density concrete[2]	20	40	50	60	70

[1] Concrete with a density in the range 2 200 to 2 500 kg/m^3.
[2] Concrete with a density < 2 000 kg/m^3 made with low density aggregate.

7.2 Maximum clear spacing of reinforcement

$$\text{clear spacing (in mm)} = \frac{47000}{f_s} \quad \text{but} \leq 300\,\text{mm} \tag{7-1}$$

where f_s is the stress in the reinforcement (in MPa) under service loads and is given by

$$f_s = 0.87 f_y \times \left[\frac{\gamma_1 + \gamma_2}{\gamma_3 + \gamma_4}\right] \times \frac{A_{s,req}}{A_{s,prov}} \times \frac{1}{\beta_b} \tag{7-2}$$

with γ_1 = partial safety factor for self-weight loads at SLS (typically = 1.1)

γ_2 = partial safety factor for imposed loads at SLS (typically = 1.0 or 0)

γ_3 = partial safety factor for self-weight loads at ULS (typically = 1.2 or 1.5)

γ_4 = partial safety factor for imposed loads at ULS (typically = 1.6 or 0)

Slabs: If any one of the following three conditions apply, the maximum clear spacing is the lesser of 3 d and 750 mm:

(a) For high yield strength steel (f_y = 450 MPa) the slab depth $h \leq 200$ mm.

(b) For mild steel (f_y = 250 MPa) the slab depth $h \leq 250$ mm.

(c) $\rho \leq 0.3\,\%$ where $\rho = \dfrac{100\,A_s}{b\,d}$

If neither of the above conditions apply, the maximum clear spacings given in Table 15 are adjusted as follows:

- If $\rho \geq 1$ %, maximum clear spacing is taken from Table 15
- If $\rho < 1$ %, maximum clear spacing is the value from Table 15 divided by ρ.

Table 15: Maximum clear spacing (in mm) between bars (SABS 0100).

Characteristic strength of reinforcement f_y (MPa)	Percentage redistribution to or from section considered*										
	−30	−25	−20	−15	−10	0	+10	+15	+20	+25	+30
250	215	230	245	260	275	300	300	300	300	300	300
450	120	130	135	145	155	170	185	195	205	210	220
485	110	120	125	135	140	155	170	180	190	195	205

* If the percentage redistribution is unknown, a value of –15 should be assumed for moments at supports and zero for span moments.

7.3 Minimum spacing of reinforcement

Table 16: Minimum clear spacing between bars.

Configuration of bars	Orientation	Minimum clear spacing
Single bars	Horizontal	$(h_{agg} + 5\text{ mm})$
	Vertical	$\frac{2}{3}h_{agg}$
Bars in pairs	Horizontal	$(h_{agg} + 5\text{ mm})$
	Vertical	$\frac{2}{3}h_{agg}$ for bars in the pair on top of each other $(h_{agg} + 5\text{ mm})$ for bars in the pair side by side
Bundled bars	Horizontal and vertical	$(h_{agg} + 15\text{ mm})$

7.4 Minimum area of reinforcement (See Table 17.)

7.5 Maximum area of reinforcement

$$\frac{100\,A_s}{b\,h} \text{ or } \frac{100\,A_{sc}}{b\,h} \leq 4\,\% \tag{7-3}$$

Where bars are being lapped the sum of the reinforcement diameters in a particular layer should not be greater than 40 % of the section width at that level. For columns the following limits apply:

$$\frac{100\,A_{sc}}{b\,h} \leq \begin{cases} 6\text{ \% for columns cast vertically} \\ 8\text{ \% for columns cast horizontally} \\ 10\text{\% at laps for both of the cases above} \end{cases} \tag{7-4}$$

Table 17: Minimum percentages of reinforcement (SABS 0100).

Situation	Definition	f_y = 250 MPa	f_y = 450 MPa
Tension reinforcement			
Sections subjected mainly to pure tension	$100A_s/A_c$	0.8	0.45
Sections subjected to flexure			
(a) Flanged beams, web in tension			
(1) $b_w/b < 0.4$	$100A_s/b_wh$	0.32	0.18
(2) $b_w/b \geq 0.4$	$100A_s/b_wh$	0.24	0.13
(b) Flanged beams, flange in tension over a continuous support			
(1) T-beam	$100A_s/b_wh$	0.48	0.26
(2) L-beam	$100A_s/b_wh$	0.36	0.20
(c) Rectangular section (in solid slabs this reinforcement should be provided in both directions)	$100A_s/A_c$	0.24	0.13
Compression reinforcement (where such reinforcement is required for the ultimate limit state)			
General rule	$100A_{sc}/A_{cc}$	0.4	0.4
Simplified rule for particular cases:			
(a) Rectangular column or wall	$100A_{sc}/A_c$	0.4	0.4
(b) Flanged beam:			
(1) Flange in compression	$100A_{sc}/bh_f$	0.4	0.4
(2) Web in compression	$100A_{sc}/b_wh$	0.2	0.2
(c) Rectangular beam	$100A_{sc}/A_c$	0.2	0.2
Transverse reinforcement in flanges of flanged beams (provided over the full effective flange width near top surface to resist horizontal shear)	$100A_{st}/h_f\ell$	0.15	0.15

A_c =	total area of concrete	b =	width of section
A_{cc} =	total area of concrete in compression	b_w =	width, or effective width of the rib*
A_{sc} =	minimum area of compression reinf.	h =	total depth of section
A_s =	minimum area of tension reinf.	h_f =	depth of flange
A_{st} =	minimum area of transverse reinf. in the flange	ℓ =	span of beam

*For a box, T-, or I-section, b_w is taken as the average width below the flange.

7.6 Reinforcement at sides of beams exceeding 750 mm in depth

$$\phi_1 \geq \sqrt{\frac{s_b\, b_w}{f_y}} \qquad (7\text{-}5)$$

where $s_b \leq 250$ mm.

Use $b_w \geq 500$ mm in above equation.

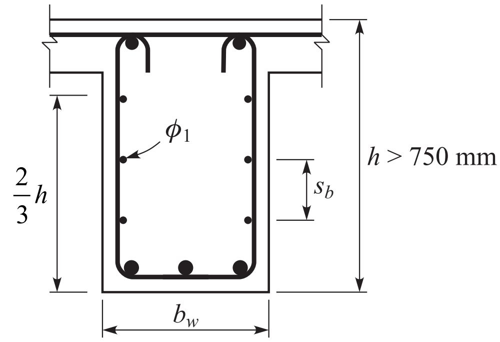


Figure 13: Reinforcement at sides of beams exceeding 750 mm in depth.

7.7 Span-effective depth ratio

Table 19: Basic span-effective depth (L/d) ratios for beams.

Support conditions	Rectangular section	Flanged section $b_w \leq 0.3b$
Truly simply supported	16	12.8
Simply supported with nominally restrained ends	20	16.0
One end continuous	24	19.2
Both ends continuous	28	22.4
Cantilevers	7	5.6

Table 18: L/d modification factors for compression reinforcement.

$\frac{100\, A'_{s,prov}}{b\, d}$	Modification factor
0.00	1.00
0.15	1.05
0.25	1.08
0.35	1.10
0.50	1.14
0.75	1.20
1.00	1.25
1.25	1.29
1.50	1.33
1.75	1.37
2.00	1.40
2.5	1.45
$\geq$ 3.0	1.50
Intermediate values may be determined by interpolation	

Modification factors for tension reinforcement:

$$MF_{A_s} = 0.55 + \frac{477 - f_s}{120\left(0.9 + \frac{M}{b d^2}\right)} \leq 2.0 \qquad (7\text{-}6)$$

Modification factor for compression reinforcement:

$$MF_{A'_s} = \left(1 + \frac{\rho'}{3 + \rho'}\right) \leq 1.5 \qquad (7\text{-}7)$$

where $$\rho' = \frac{100\, A'_{s,prov}}{b\, d} \qquad (7\text{-}8)$$

If the basic L/d ratio is applied to a beam with span longer than 10 m, the basic L/d ratio should be multiplied by $10/L$

Table 20: *L*/*d* modification factors for tension reinforcement.

Steel service stress f_s (MPa)	M / bd^2											
	0.5	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
300	1.60	1.33	1.16	1.06	0.98	0.93	0.89	0.85	0.82	0.80	0.78	0.76
290	1.66	1.37	1.20	1.09	1.01	0.95	0.90	0.87	0.84	0.81	0.79	0.78
280	1.72	1.41	1.23	1.12	1.03	0.97	0.92	0.89	0.85	0.83	0.81	0.79
270	1.78	1.46	1.27	1.14	1.06	0.99	0.94	0.90	0.87	0.84	0.82	0.80
260	1.84	1.50	1.30	1.17	1.08	1.01	0.96	0.92	0.88	0.86	0.83	0.81
250	1.90	1.55	1.34	1.20	1.11	1.04	0.98	0.94	0.90	0.87	0.85	0.82
240	1.96	1.59	1.37	1.23	1.13	1.06	1.00	0.95	0.92	0.88	0.86	0.84
230	2.00	1.63	1.41	1.26	1.16	1.08	1.02	0.97	0.93	0.90	0.87	0.85
220	2.00	1.68	1.44	1.29	1.18	1.10	1.04	0.99	0.95	0.91	0.88	0.86
210	2.00	1.72	1.48	1.32	1.20	1.12	1.06	1.00	0.96	0.93	0.90	0.87
200	2.00	1.76	1.51	1.35	1.23	1.14	1.07	1.02	0.98	0.94	0.91	0.88
190	2.00	1.81	1.55	1.37	1.25	1.16	1.09	1.04	0.99	0.96	0.92	0.90
180	2.00	1.85	1.58	1.40	1.28	1.18	1.11	1.06	1.01	0.97	0.94	0.91
170	2.00	1.90	1.62	1.43	1.30	1.21	1.13	1.07	1.02	0.98	0.95	0.92
160	2.00	1.94	1.65	1.46	1.33	1.23	1.15	1.09	1.04	1.00	0.96	0.93
150	2.00	1.98	1.69	1.49	1.35	1.25	1.17	1.11	1.05	1.01	0.98	0.94
140	2.00	2.00	1.72	1.52	1.38	1.27	1.19	1.12	1.07	1.03	0.99	0.96
130	2.00	2.00	1.75	1.55	1.40	1.29	1.21	1.14	1.09	1.04	1.00	0.97
120	2.00	2.00	1.79	1.58	1.43	1.31	1.23	1.16	1.10	1.05	1.01	0.98

8 Design of Beams

8.1 Effective span length

Table 21: Effective lengths in beams.

Beam	Effective span length
Simply supported	The lesser of: • distance between centres of bearings, and • clear distance between supports plus an effective depth.
Continuous beam	Distance between centres of supports. For an embedded end the centre of support should be taken as half an effective depth from the face of the support.
Cantilever	The length to the face of the support plus half an effective depth. If the cantilever forms part of a continuous beam the effective length should be taken as the clear length plus the distance to the centre of the support.

8.2 Analysis of continuous beams

Conditions for the use of the simplified method:

(a) $Q_n \leq 1.25\, G_n$

(b) The loads on the beam must be substantially uniformly distributed loads.

(c) There must be 3 or more spans.

(d) The spans may not vary by more than 15% in length with regard to the longest span.

Table 22: Ultimate bending moments and shear forces for continuous beams (simplified method).

Position	Moment	Shear
Outer support	0	$0.45F$
Near centre of end span	$F\,L_{eff}\,/11$	-
First internal support	$-F\,L_{eff}\,/9$	$0.6F$
Centre of interior span	$F\,L_{eff}\,/14$	-
Interior support	$-F\,L_{eff}\,/12$	$0.55F$
These moments may not be redistributed. Assume β_b = 0.9. F = Total load on span (in kN) = 1.2 G_n + 1.6 Q_n L_{eff} = Effective span		

8.3 Flanged beams

Effective flange width:

T-section $$b_{eff} = \text{lesser of} \begin{cases} b_w + \dfrac{L_z}{5} \\ \text{actual flange width} \end{cases} \tag{8-1}$$

L-section $$b_{eff} = \text{lesser of} \begin{cases} b_w + \dfrac{L_z}{10} \\ \text{actual flange width} \end{cases} \tag{8-2}$$

where L_z is the distance between zero moments. As a simplified approach for continuous beams, L_z can be assumed to be 0.7 of the effective span.

8.4 Beams with compression reinforcement

$$\left(A'_{s,prov} - A'_{s,req}\right) \geq \left(A_{s,prov} - A_{s,req}\right) \tag{8-3}$$

Containment of compression reinforcement:

- Links should pass around outer bars and each alternate bar
- The link should be at least 1/4 the size of the largest compression bar
- The maximum longitudinal spacing of links is 12 times the diameter of the smallest compression bar
- For the containment to be effective, the link should pass around the bar with an inside angle not more than 135º
- No compression bar should not be more than 150 mm from a contained bar

8.5 Curtailment of reinforcement

The curtailment anchorage length must be the greater of:

1. the effective depth d of a member, or
2. twelve times the bar size (12 ϕ).

For bars in tension, the smallest distance from one of the following conditions must also apply:

3. The bars must extend the ultimate anchorage bond length L_{ua} beyond the theoretical cut-off point (TCP). For the ultimate anchorage bond the stress in the bar is taken as 0.87 f_y.
4. At the physical cut-off point (PCP) the shear capacity is twice the actual shear force.
5. At the PCP the flexural capacity of remaining bars is twice the actual bending moment.

If the conditions to the use of Table 22 applies, the simplified rules for curtailment may be used.

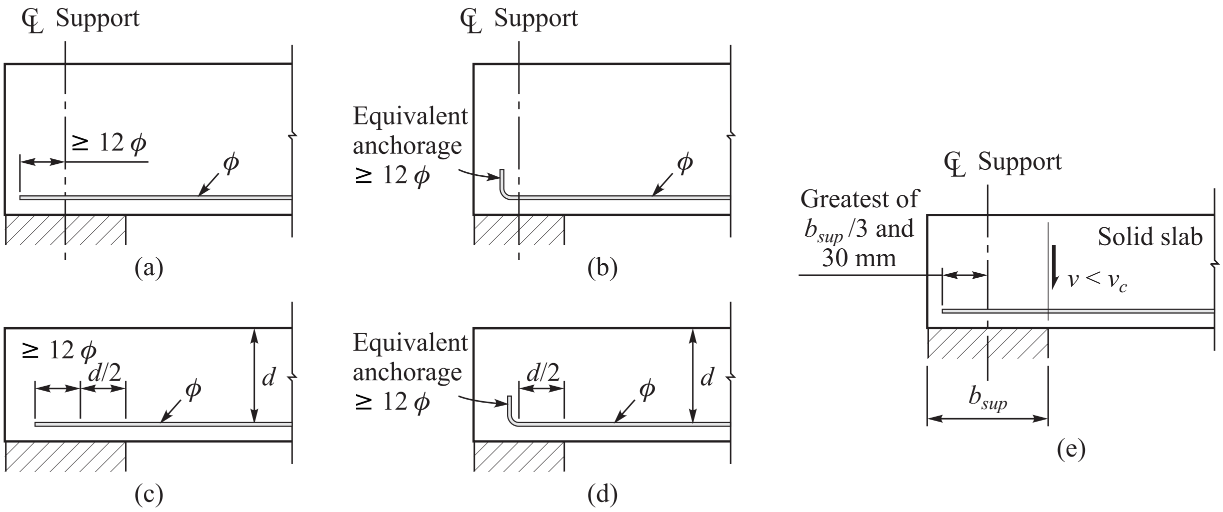


Figure 14: Curtailment of reinforcement–simply supported end.

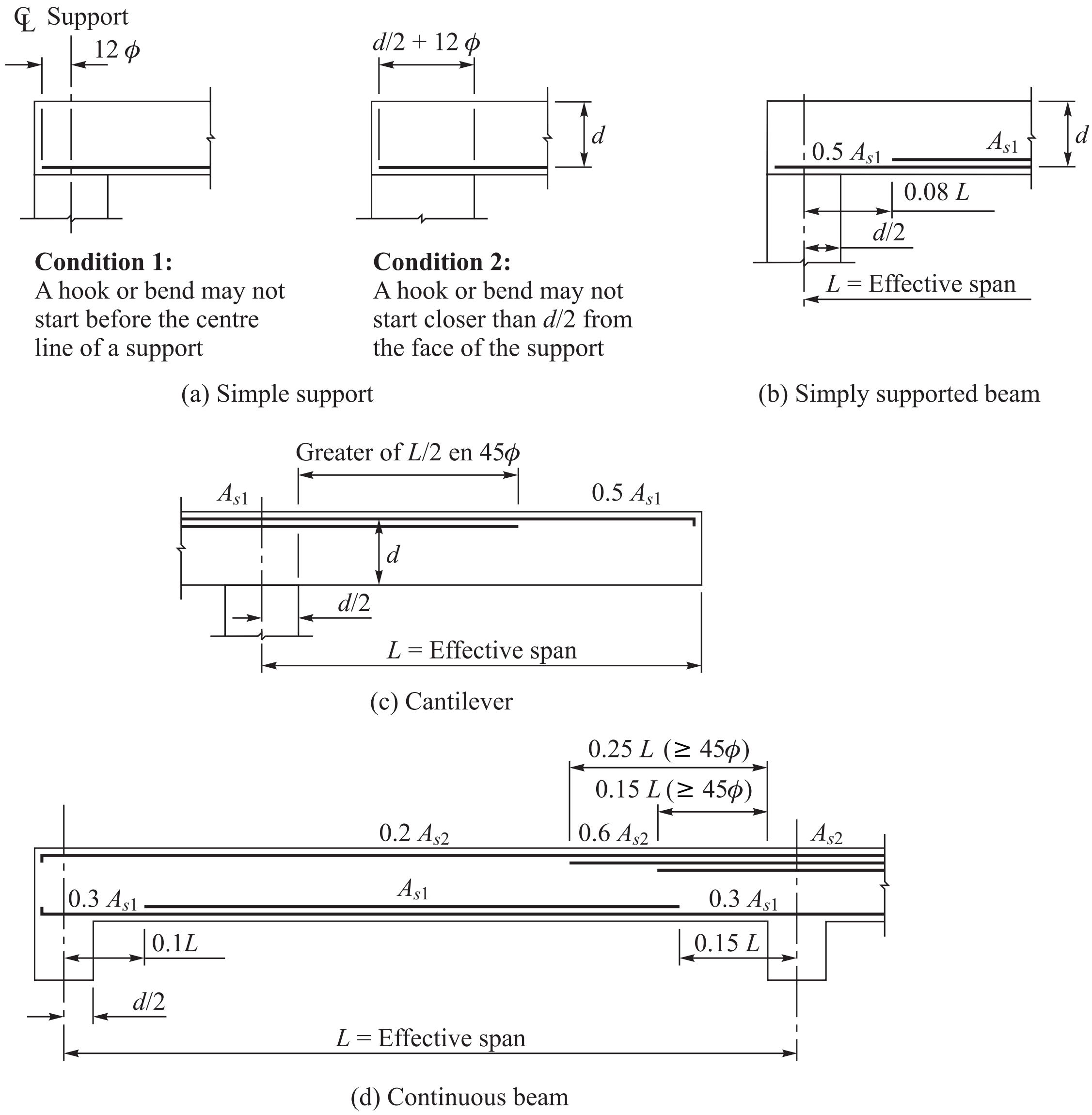


Figure 15: Simplified curtailment rules for beams.

9 Design of Short Columns

9.1 Moment-axial force interaction diagram

Stress and strain in reinforcement

$$f_s(\varepsilon_s) = \begin{cases} 0.87 f_y & \text{for } \varepsilon_s \le \varepsilon_y \\ E_s \varepsilon_s & \text{for } \varepsilon_y < \varepsilon_s < \varepsilon_{yc} \\ f_{yc} & \text{for } \varepsilon_s \ge \varepsilon_{yc} \end{cases} \quad (9\text{-}1)$$

$$\varepsilon_s = -0.0035 \left(\frac{d - x}{x} \right) \quad (9\text{-}2)$$

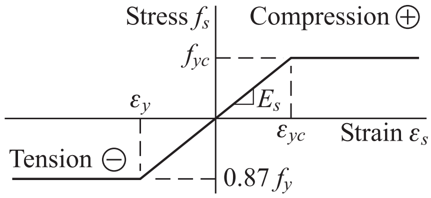


Figure 16: Sign convention

From equilibrium:

$$N = 0.45 f_{cu}\, b s + f_{sc} A'_s + f_s A_s \quad (9\text{-}3)$$

$$M = 0.45 f_{cu}\, b s \left(\bar{x}_p - \frac{s}{2} \right) + f_{sc} A'_s \left(\bar{x}_p - d' \right) - f_s A_s \left(d - \bar{x}_p \right) \quad (9\text{-}4)$$

Moments about top of section yields plastic centroid: $\bar{x}_p = \dfrac{F_{cc}(h/2) + F_{sc} d' + F_s d}{F_{cc} + F_{sc} + F_s}$ (9-5)

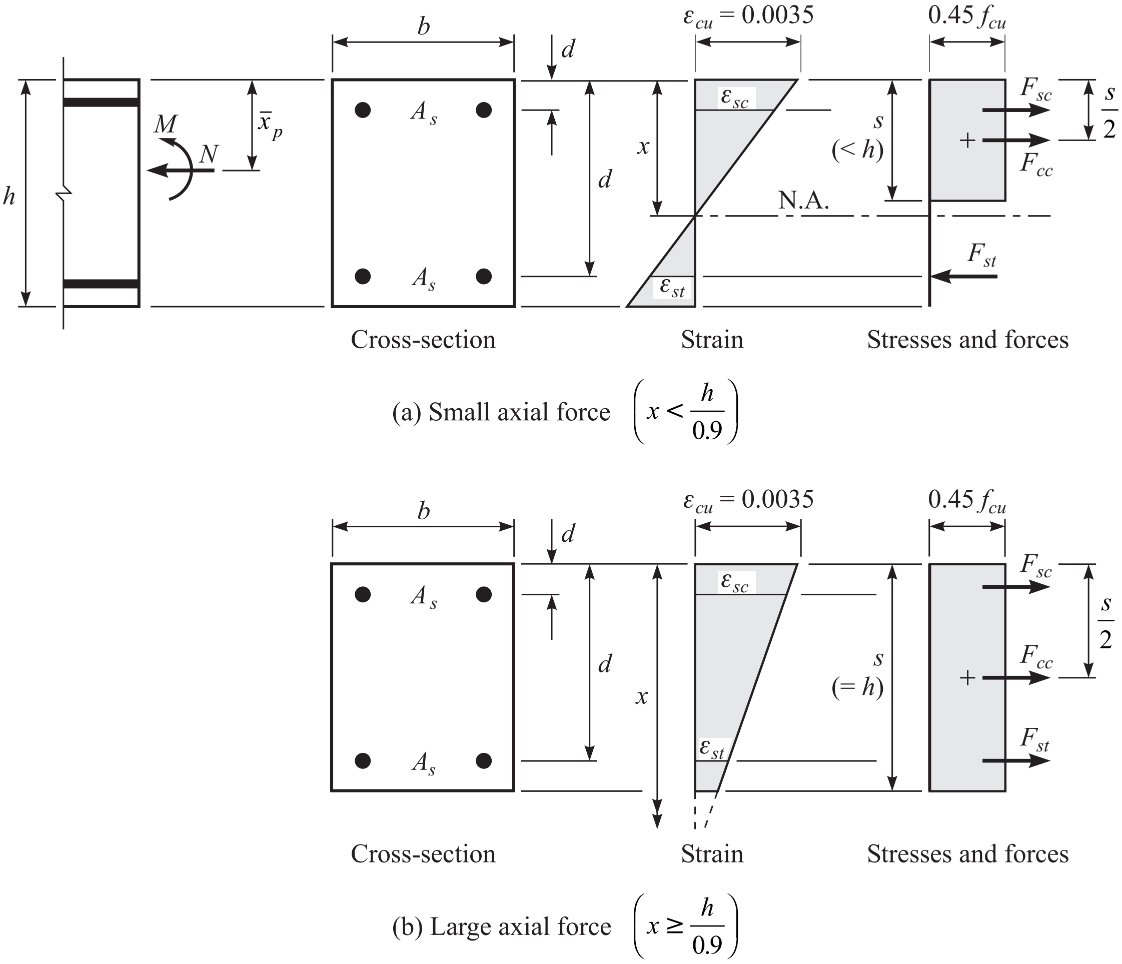


Figure 17: Moment and axial force acting on a section.

Defining points:

(a) Pure flexure, $N = 0$

(b) Balance point $\varepsilon_s = \varepsilon_y$

$$x_{bal} = \frac{d}{1 + \dfrac{|\varepsilon_y|}{0.0035}} \qquad \text{(9-6)}$$

(c) Pure compression $M = 0$

(d) $F_{st} = 0$, $\varepsilon_s = 0$, $x = d$

(e) Compression reinforcement yields $\varepsilon_{sc} = \varepsilon_{yc}$. Point (e) can be above or below (b).

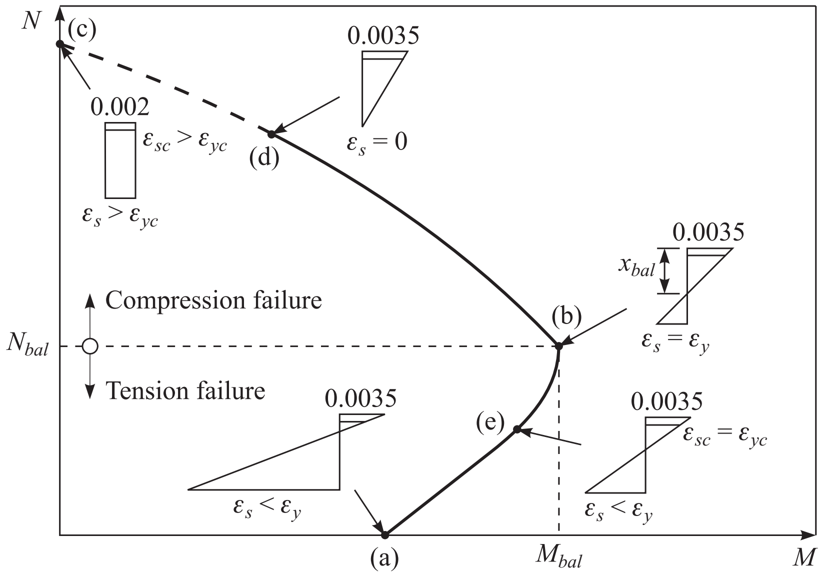


Figure 18: Points defining the *MN*-interaction diagram.

9.2 Axially loaded short column

Minimum eccentricity $e_{min} = 0.05\ h \leq 20$ mm (h is measured perpendicular to axis of bending).

Nominal eccentricity moment $\qquad M_{min} = N\ e_{min} \qquad$ (9-7)

For moments less than M_{min} the axial capacity is $\qquad N_u = 0.40 f_{cu} A_c + 0.67 f_y A_{sc} \qquad$ (9-8)

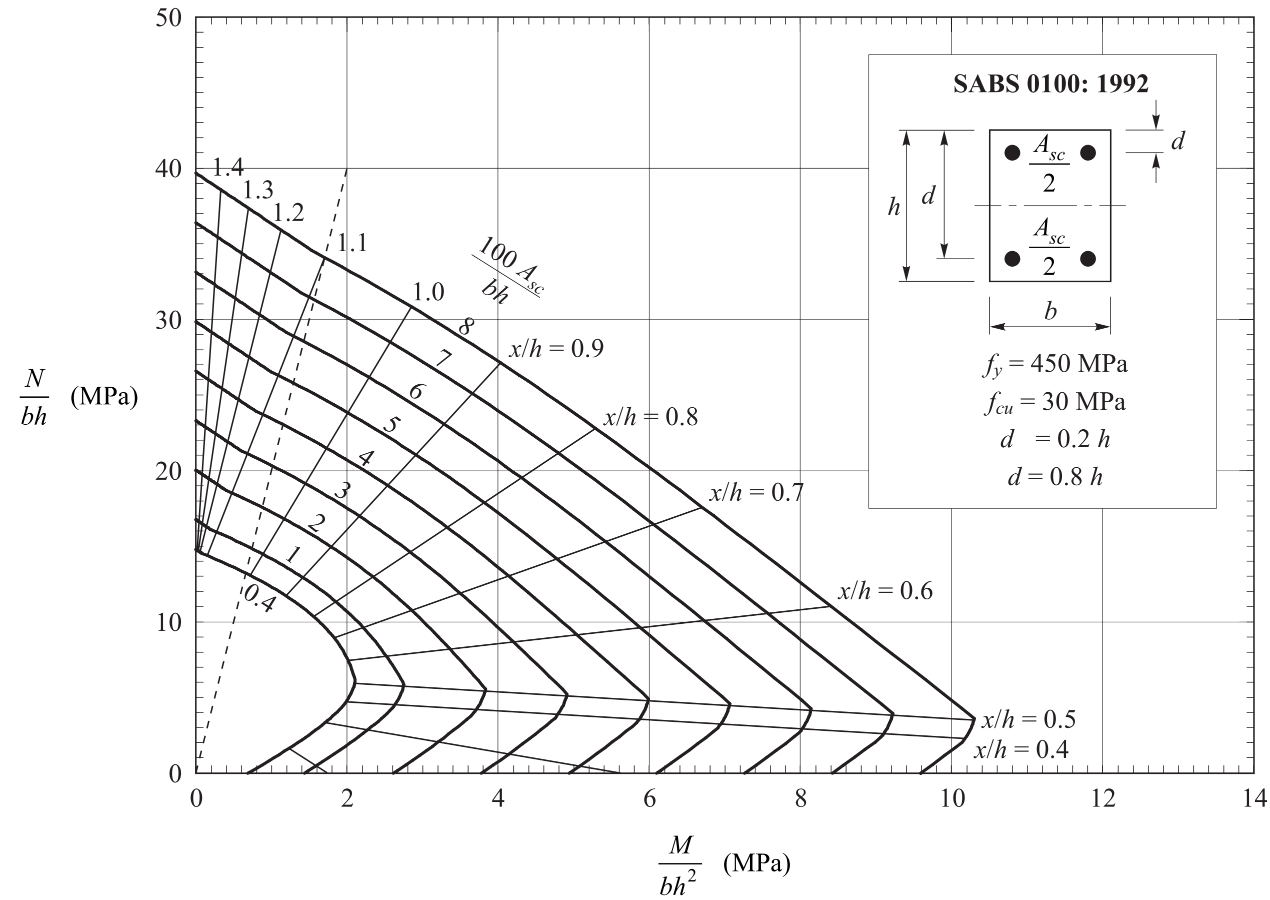


Figure 19: Typical *MN*-interaction diagram.

10 Design of Suspended Floors

10.1 One-way spanning slabs

The slab should be designed to span in one direction if the long span exceeds 3 times the short span. The single load case of maximum design load on all spans may be used when:

(a) area of a bay $\geq$ 30 m^2

(b) $Q_n \leq 1.25 G_n$ (10-1)

(c) $Q_n \leq 5$ kPa (10-2)

(d) Reinforcement must be curtailed according to the simplified rules (Fig. 20).

When using this single load case in the analysis of a continuous slab, a redistribution of moments should be applied by reducing the support moments by 20% and increasing the span moments accordingly. Nowhere should the redistributed moments be less than 70% of the elastic moments.

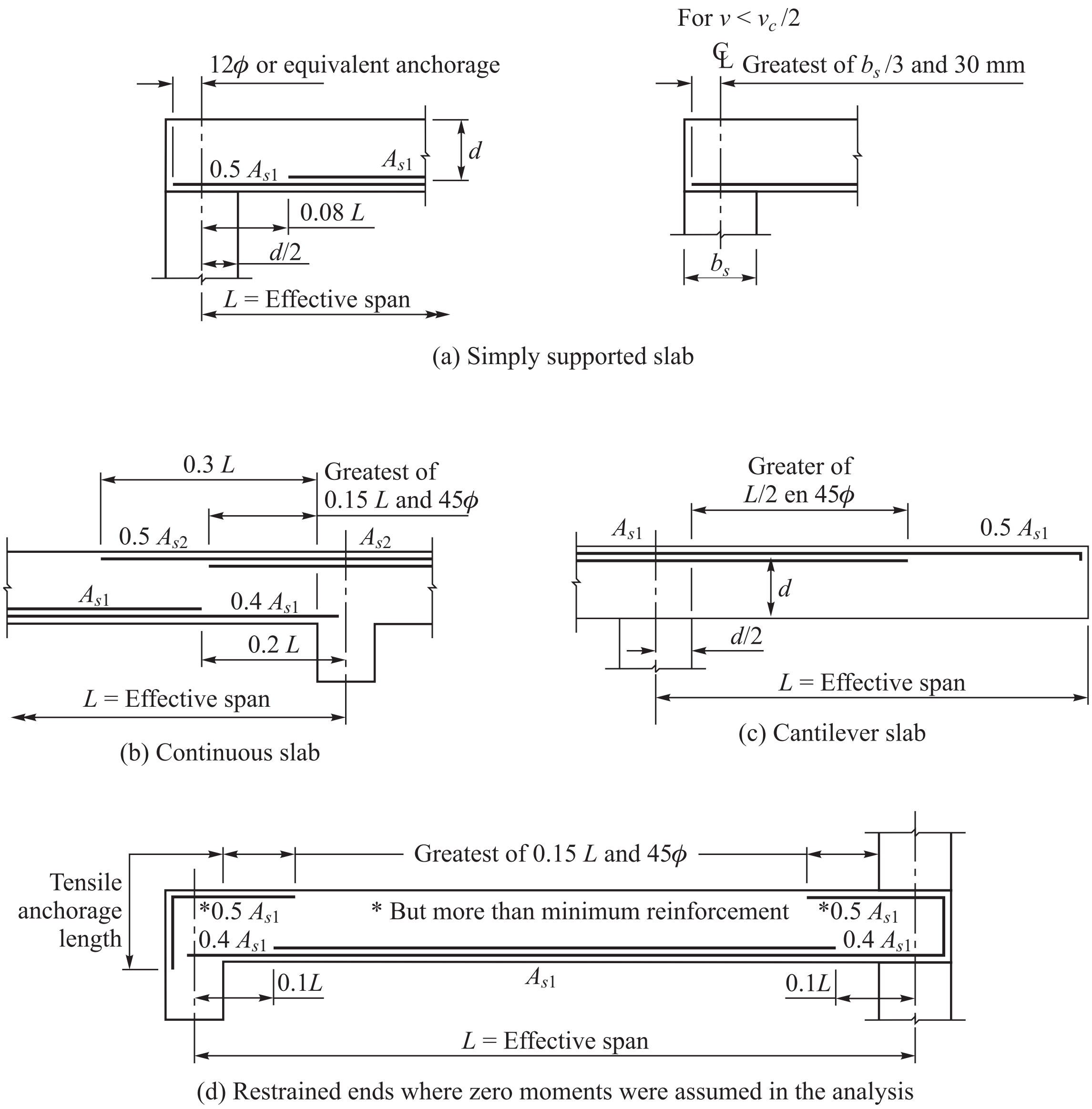


Figure 20: Simplified detailing rules for one-way spanning slabs.

If the span adjacent to a cantilever is less than 3 times the length of the cantilever, the load case where the cantilever carries the maximum load and the adjacent span carrying a minimum load should also be considered.

The simplified method given in Table 23 may used if:

(a) The above conditions for the simplified load arrangement apply

(b) The spans are approximately equal

(c) There are 3 or more spans.

Table 23: Ultimate bending moments and shear forces in one-way spanning slabs.

Position	Moment	Shear
Outer support:	0	$0.4F$
Near centre of end span	$0.086\ F\ L_{eff}$	-
First interior support	$-0.086\ F\ L_{eff}$	$0.6F$
Centre of interior span	$0.063\ F\ L_{eff}$	-
Interior support	$-0.063\ F\ L_{eff}$	$0.5F$
These values may not be redistributed. $F = 1.2\ G_n + 1.6\ Q_n$, L_{eff} = Effective span		

Concentrated loads

The width of beam supporting the load is the sum of the load width and the following width on each side of the load

$$1.2x\left(1-\frac{x}{L}\right) \leq 0.3L \qquad (10\text{-}3)$$

where x is measured from the nearest support to the load.

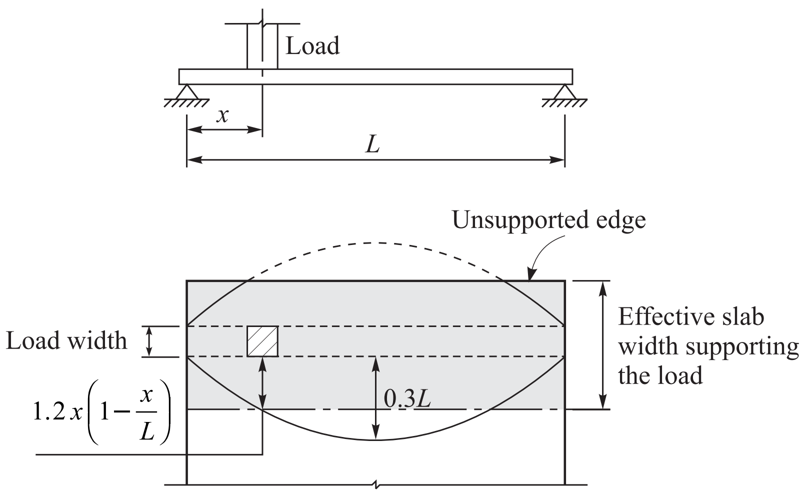


Figure 21: Effective width of slab supporting a concentrated load.

10.2 Two-way spanning edge supported slabs

10.2.1 Simply supported slabs

For a rectangular slab, simply supported along all four edges so that lifting of the corners are not prevented, the maximum moments per unit width in the centre of the slab is given by

$$m_{sx} = \alpha_{sx}\, n\, \ell_x^2$$
$$m_{sy} = \alpha_{sy}\, n\, \ell_x^2 \qquad (10\text{-}4a, b)$$

where ℓ_x = short span

ℓ_y = long span

n = total design load (kN/m^2)

= $1.2\ g_n + 1.6\ q_n$

Detailing rules are given below.

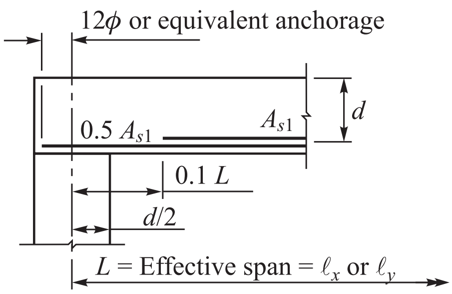


Figure 22: Simplified detailing rules for two-way spanning simply supported slabs.

Table 24: Bending moment coefficients for simply supported two-way spanning slabs.

ℓ_y / ℓ_x	α_{sx}	α_{sy}
1.0	0.045	0.045
1.1	0.061	0.038
1.2	0.071	0.031
1.3	0.080	0.027
1.4	0.087	0.023
1.5	0.092	0.020
1.6	0.097	0.017
1.7	0.100	0.015
1.8	0.102	0.016
1.9	0.103	0.016
2.0	0.104	0.016
2.5	0.108	0.016
3.0	0.111	0.017

10.2.2 Slabs with restrained edges

The simplified method below may be used if:

(a) The nominal dead and imposed loads on adjacent panels should be approximately the same as on the panel under consideration.

(b) In the direction of the span being considered, the adjacent span lengths must be approximately equal to that of the span under consideration.

$$m_{sx} = \beta_{sx}\, n\, \ell_x^2$$
$$m_{sy} = \beta_{sy}\, n\, \ell_x^2 \qquad (10\text{-}5a, b)$$

where β_{sx} and β_{sy} are bending moment coefficients from Table 25.

Table 25: Bending moment coefficients for rectangular panels supported on four sides with provision for torsional reinforcement in corners.

Type of panel and moments considered	**Short span coefficients β_{sx} for ℓ_y/ℓ_x**								**Long span coefficients β_{sy} for all ℓ_y/ℓ_x**
	1.0	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.75**	**2.0**	
1. Interior panel									
Negative moment at continuous edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at midspan	0.024	0.028	0.032	0.036	0.039	0.041	0.045	0.049	0.024
2. One short edge discontinuous									
Negative moment at continuous edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
Positive moment at midspan	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
3. One long edge discontinuous									
Negative moment at continuous edge	0.039	0.049	0.056	0.062	0.068	0.073	0.082	0.089	0.037
Positive moment at midspan	0.030	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.028
4. Two adjacent edges discontinuous									
Negative moment at continuous edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.092	0.045
Positive moment at midspan	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.034
5. Two short edges discontinuous									
Negative moment at continuous edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	-
Positive moment at midspan	0.034	0.038	0.040	0.043	0.045	0.045	0.047	0.053	0.034
6. Two long edges discontinuous									
Negative moment at continuous edge	-	-	-	-	-	-	-	-	0.045
Positive moment at midspan	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
7. Three edges discontinuous (One long edge continuous)									
Negative moment at continuous edge	0.057	0.065	0.071	0.076	0.080	0.084	0.092	0.098	-
Positive moment at midspan	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.044
8. Three edges discontinuous (One long edge continuous)									
Negative moment at continuous edge	-	-	-	-	-	-	-	-	0.058
Positive moment at midspan	0.042	0.054	0.063	0.071	0.078	0.084	0.096	0.105	0.044
9. Four edges discontinuous									
Positive moment at midspan	0.055	0.065	0.074	0.081	0.087	0.092	0.103	0.111	0.056

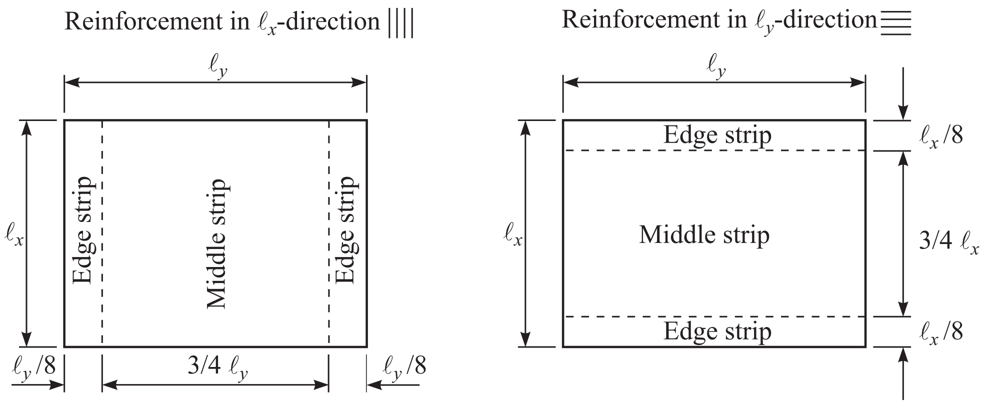


Figure 23: Distribution of slab into middle and edge strips.

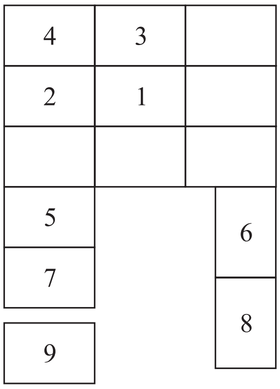


Figure 24: Support conditions for slab panels in Table 25.

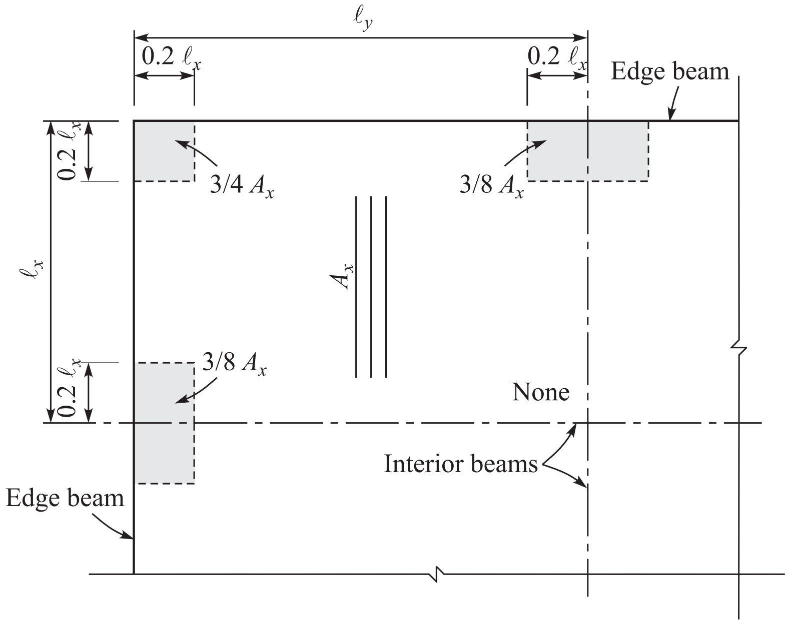


Figure 25: Placing and quantities of torsional reinforcement.

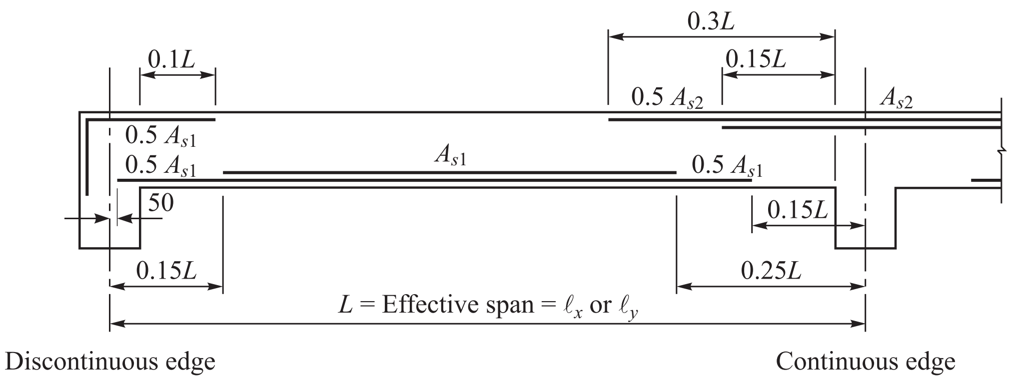


Figure 26: Simplified detailing rules for two-way spanning slabs with restrained edges.

The method illustrated in Fig. 27 is proposed when moments at the mutual support to two adjacent slabs differ.

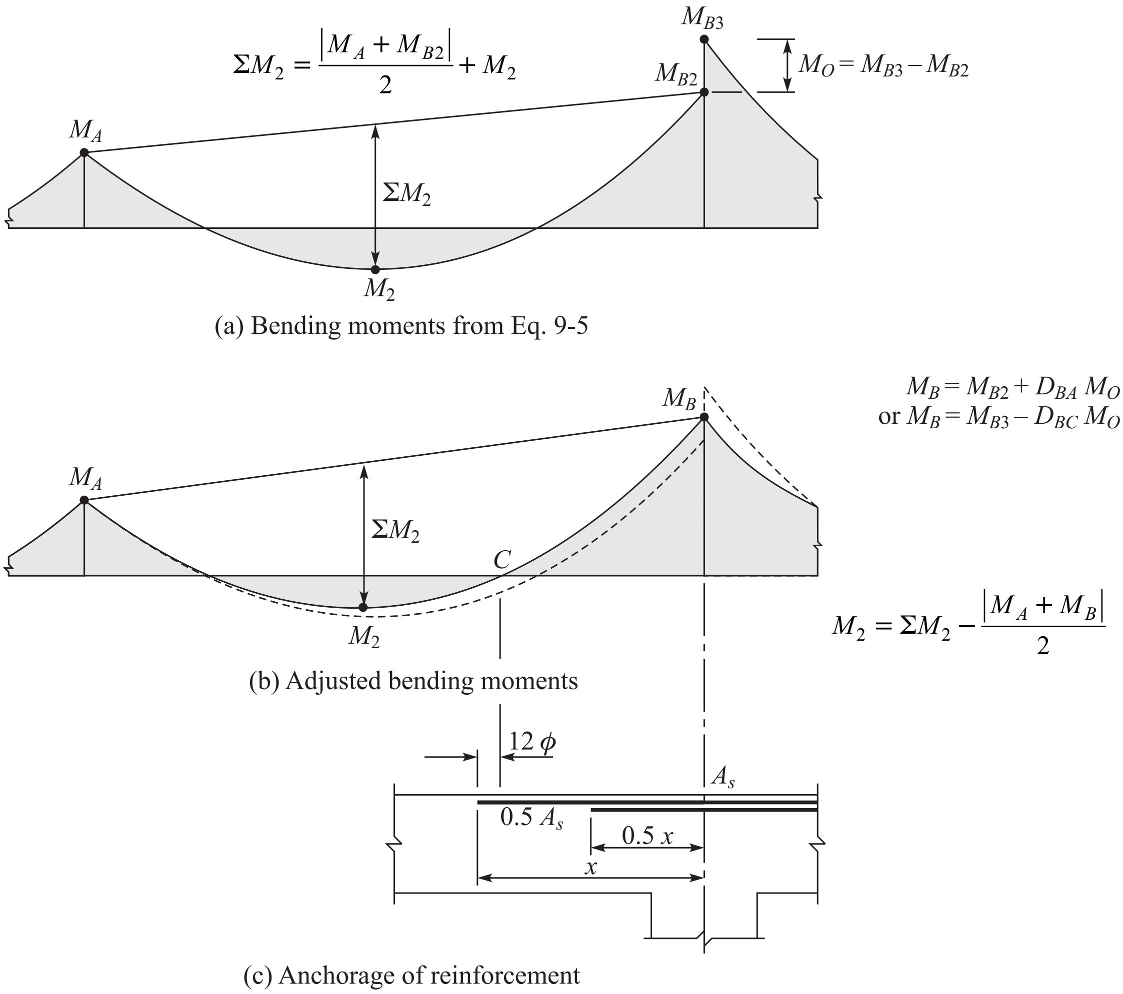


Figure 27: Adjustment to moments in edge supported slabs.

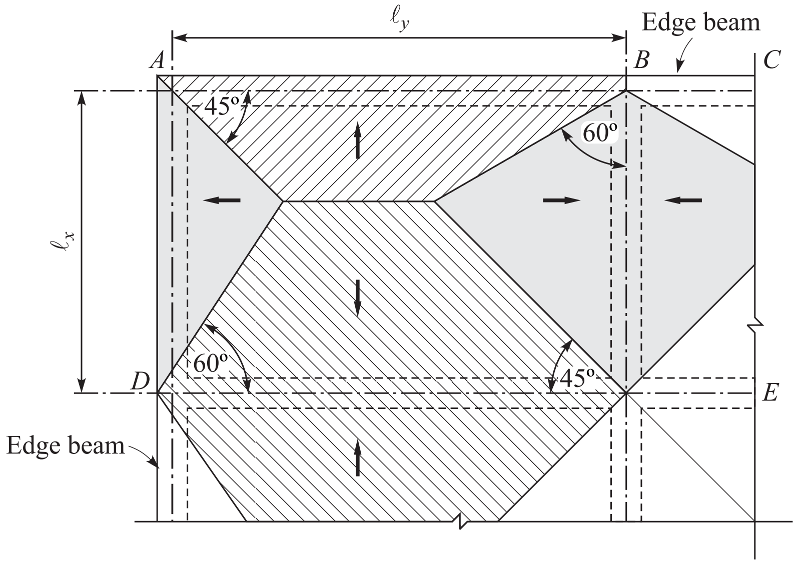


Figure 28: Loads on supporting beams.

10.3 Flat slabs

10.3.1 Notation

ℓ_1 = length of the panel, measured between column centre lines in the direction under consideration

ℓ_2 = width of the panel, measured between column centre lines perpendicular to the direction under consideration

ℓ_x = shorter span

ℓ_y = longer span

$$\ell = \text{effective span} = \left(\ell_1 - \frac{2}{3}h_c\right) \tag{10-1}$$

h_c = effective diameter of the column head

$$= \left\{\begin{matrix} \ell_h & \text{for round column heads} \\ \sqrt{\dfrac{4\,\ell_h^2}{\pi}} & \text{for square column heads} \end{matrix}\right\} \le \frac{1}{4}\left(\frac{\ell_1 + \ell_2}{2}\right) \tag{10-2}$$

ℓ_h = effective dimension of a column head

$$= \text{lesser of} \left\{\begin{matrix} \ell_{ho} \\ \ell_{h,max} = \ell_c + 2\left(d_h - 40\text{ mm}\right) \end{matrix}\right. \tag{10-3}$$

ℓ_{ho} = actual dimension of column head

d_h = depth of column head below soffit (or below drop, if present)

ℓ_c = dimension of the column (measured in the same direction)

10.3.2 Analysis

Following the analysis of an equivalent frame, the bending moments should be divided in column and middle strips (see Fig. 31) as shown in Table 26. The maximum design moment at a support can be taken as the moment a distance $h_c/2$ from the column centre line, provided the sum of the positive span moment and the average of the support moments is greater than the following

$$\frac{n\,\ell_2}{8}\left(\ell_1 - \frac{2}{3}h_c\right)^2 \tag{10-4}$$

The maximum moment that can be transferred to a column is given by

$$M_{t,max} = 0.15\, b_e\, d^2\, f_{cu} \tag{10-5}$$

where b_e = width of a strip depending on the distance between the column and the free

Table 26: Division of moments in strips (SABS 0100).

	Column strip	**Middle strip**
Negative moment	75%	25%
Positive moment	55%	45%
For the case where the width of the column strip is taken as equal to that of the drop and the width of the middle strip therefore increases, the design moments resisted by the middle strip should be increased in proportion to its increase in width. The design moments to be resisted by the column strip may then be decreased by an amount such that the total negative and total positive moments resisted by the column and middle strips together are unchanged.		

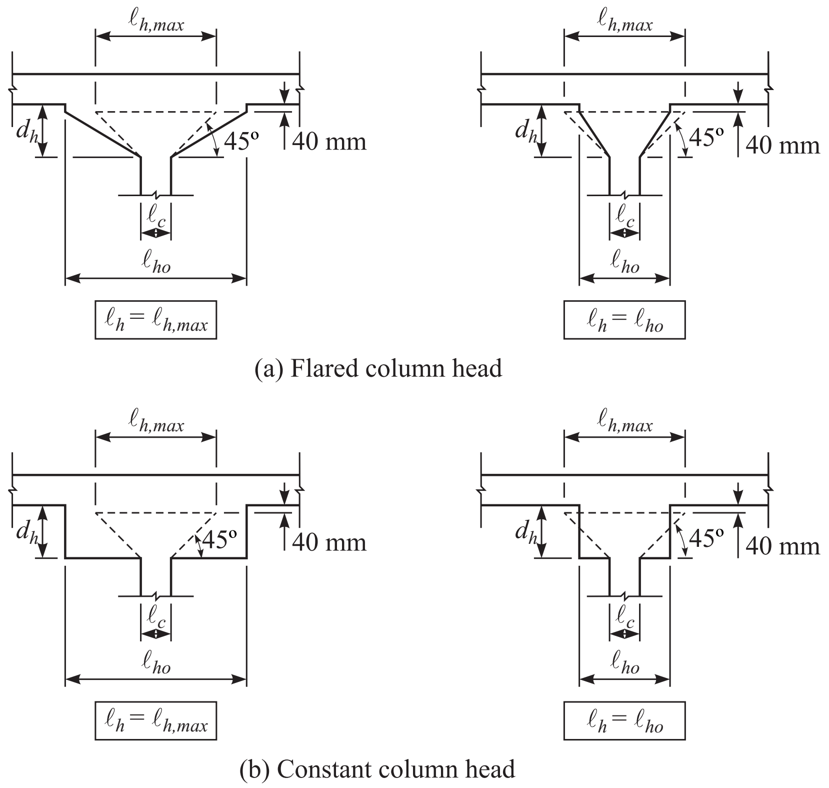


Figure 29: Effective dimension of a column head.

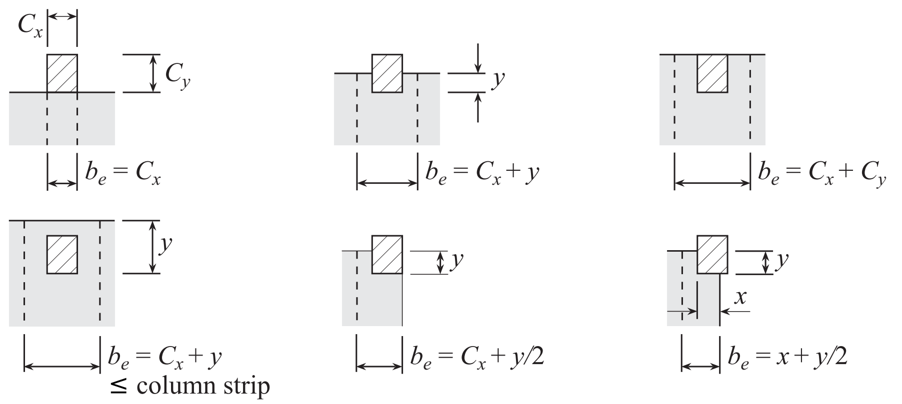


Figure 30: Width of strip b_e to transfer moment at slab-column connection.

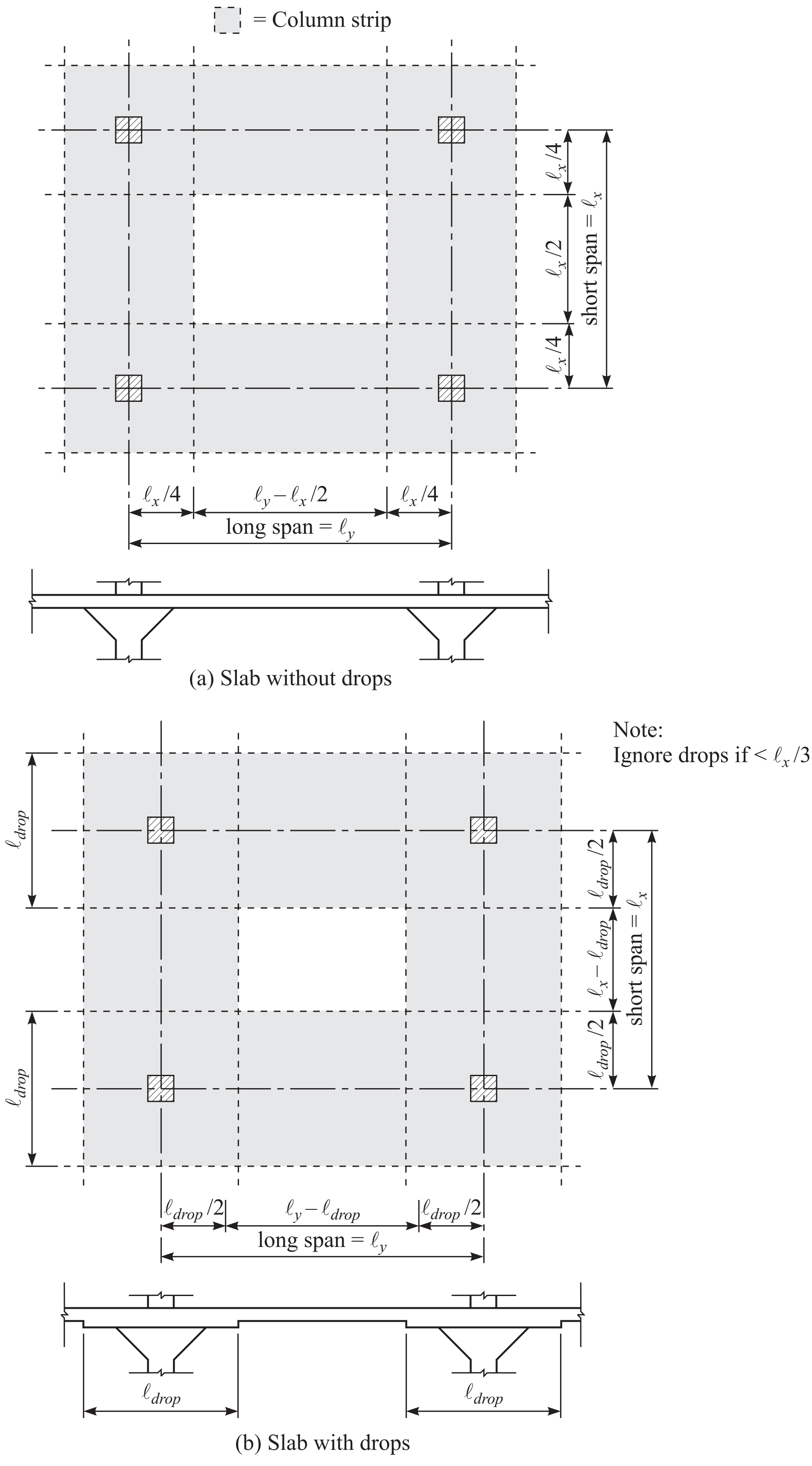


Figure 31: Division of panels into column and middle strips.

Table 27: Ultimate bending moments and shear flat slabs.

Position	Moment	Shear	Total column moment
Outer support: Column	$-0.04\ F\ell$	$0.45\ F$	$0.04\ F\ell$
Outer support: Wall	$-0.02\ F\ell$	$0.4\ F$	—
Near centre of end span	$+0.083\ F\ell$	—	—
First internal support	$-0.063\ F\ell$	$0.6\ F$	$0.022\ F\ell$
Centre of interior span	$+0.071\ F\ell$	—	—
Interior support	$-0.055\ F\ell$	$0.5\ F$	$0.022\ F\ell$
These moments may not be redistributed. Assume $\beta_b = 0.8$. F = Total load on span (in kN) = $1.2\ G_n + 1.6\ Q_n$ ℓ = Effective span in the direction under consideration $= \left(\ell_1 - \frac{2}{3} h_c\right)$			

edge of the slab (see Fig. 30). For an internal column b_e should not be greater than the width of the column strip.

d = effective depth of top reinforcement in the column strip

f_{cu} = characteristic concrete strength

If the conditions for the simplified load arrangement (see section 10.1) apply, the simplified method given in Table 27 may be used to obtain the bending moments and shear forces in the slab. The following additional requirements must also be met before the simplified method may be used:

(d) The structure is braced so that sideways stability of the frame does not depend on the slab-column connection. (The structure is braced).

(e) There should be at least three spans in the direction under consideration.

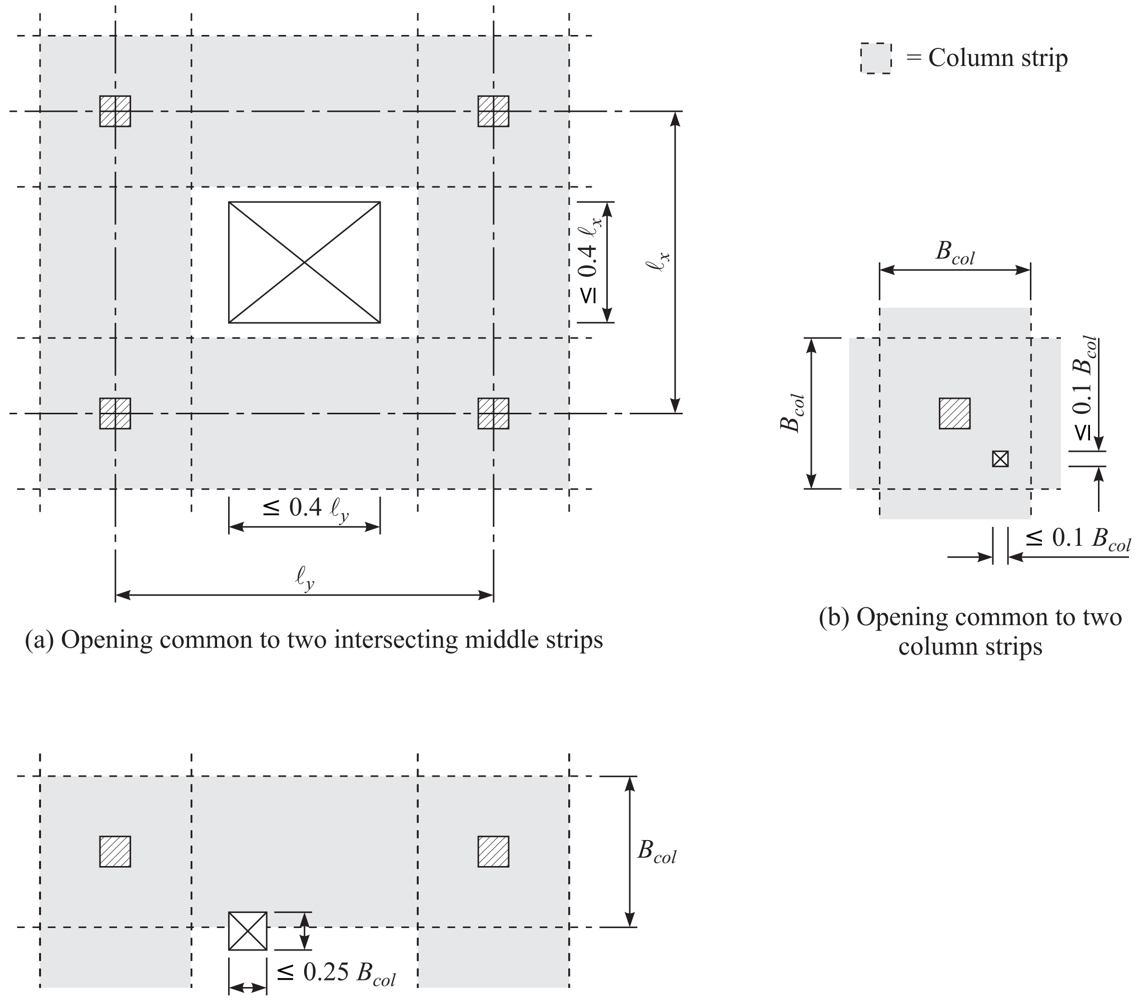
(f) The span lengths should be approximately equal. (It is assumed here that the spans will be approximately equal if they do not differ by more than 15% from the longest span).

(g) The curtailment rules for solid one-way spaning slabs (Fig. 20, sec. 10.1) should be used.

10.3.3 Deflections

For flat slabs with drops the basic allowable ℓ/d-ratio for beams is used but multiplied by a factor of 0.9. If the plan dimensions of the drops are at least a one-third of the respective span in each direction, the 0.9 factor can be omitted.

The ℓ/d-ratio should always be considered in the critical direction, which is usually the long-span direction in flat slabs.



(a) Opening common to two intersecting middle strips

(b) Opening common to two column strips

(c) Opening common to a column strip and a middle strip

Figure 32: Openings in panels.

10.3.4 Detailing of reinforcement

If the simplified method have been used to obtain the bending moments (section 10.1), the curtailment rules for solid two-way spaning slabs (see section 10.1.2) should be used.

The column strip reinforcement that passes over the column must be placed so that two-thirds of this reinforcement is placed within half the width of the column strip, centrally over the column.

10.4 Punching shear in slabs

The maximum shear stress at the edge of the loaded area should not exceed

$$v_u = \text{lesser of} \begin{cases} 0.75\sqrt{f_{cu}} \\ 4.75\,\text{MPa} \end{cases} \tag{10-6}$$

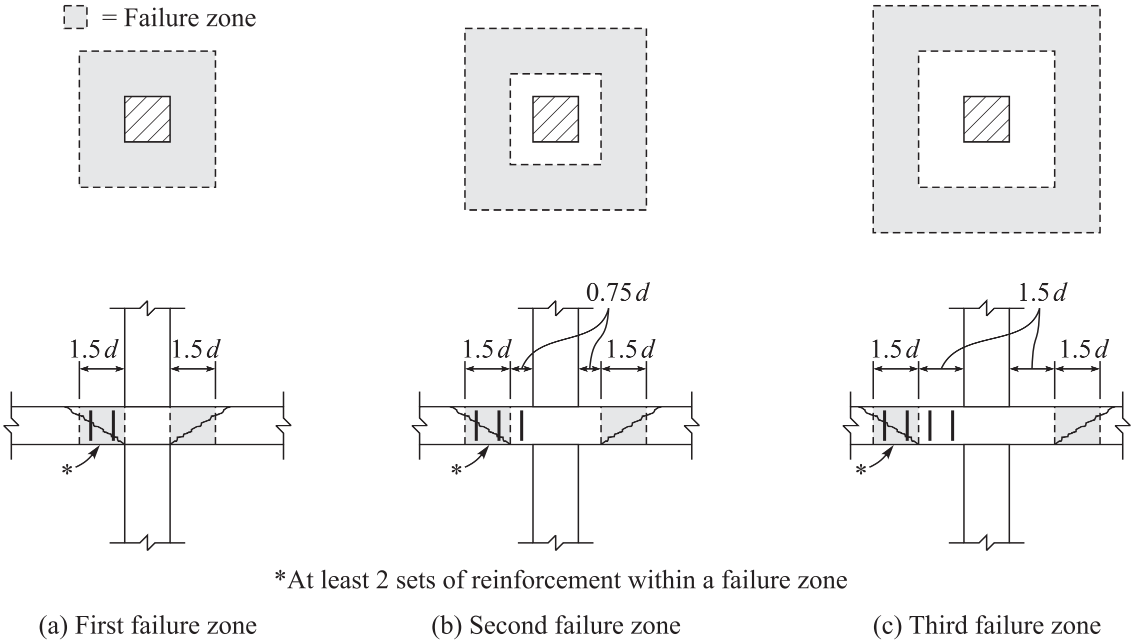


Figure 33: Considering successive failure zones for punching shear.

The design process is summarized as follows:

(a) The first perimeter is considered at a distance 1.5 d from the loaded area. If $v \leq v_c$ no shear reinforcement is required and no further checks are necessary. Reinforcement used for v_c must extend at least an effective depth d or 12 diameters beyond the zone on either side.

(b) If $v > v_c$ the following shear reinforcement is required:

$$\text{For } \left(v_c < v < 1.6\,v_c\right): \quad \Sigma A_{sv} \geq \frac{\left(v - v_c\right) u\, d}{0.87\, f_{yv}}$$

$$\text{with } \left(v - v_c\right) \geq 0.4 \text{ MPa} \qquad (10\text{-}4)$$

$$\text{For } \left(1.6\,v_c < v < 2\,v_c\right): \quad \Sigma A_{sv} \geq \frac{5\left(0.7\,v - v_c\right) u\, d}{0.87\, f_{yv}}$$

where:

u = outside perimeter of the failure zone

A_{sv} = area of shear reinforcement

f_{yv} = characteristic strength of the shear reinforcement (not exceeding 450 MPa)

These equations apply only when:

- $v < 2\,v_c$
- Links are used as shear reinforcement.
- The slab is at least 200 mm thick. For every 10 mm less than 200 mm, a 10 % loss of efficiency should be assumed for the shear reinforcement.

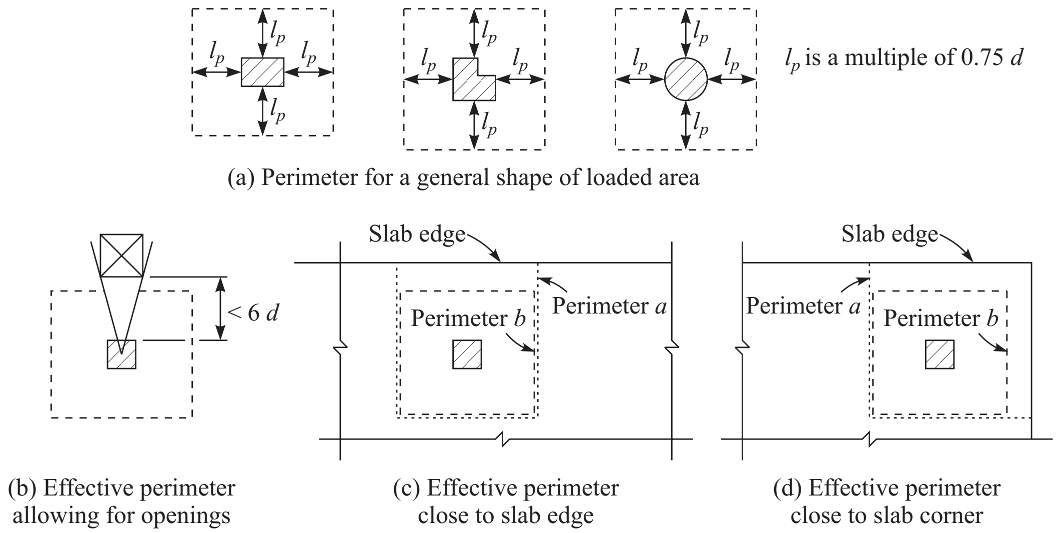


Figure 34: Perimeter for punching shear.

(c) Shear reinforcement must be distributed into at least two perimeters within the failure zone under consideration (see Fig. 3-33).

(d) Shear reinforcement placed for a previous failure zone may be included in the failure zone under consideration where such zones overlap.

(e) The first perimeter of shear reinforcement should be approximately 0.5 d from the face of the loaded area and should contain at least 40 % of the required area of shear reinforcement.

(f) Shear reinforcement must be anchored around at least one layer of tension reinforcement.

(g) Shear stresses are checked on the next perimeter a distance 0.75 d from the current perimeter. If $v \leq v_c$ no shear reinforcement is required and no further checks are necessary, otherwise repeat the process from step (b) above.

The effective shear forces in flat slabs can be calculated from Figs. 36 and 35. The simplified equations (shown with *) may be used when the structure is braced, the ratio of spans does not exceed 1.25 and the maximum design load is applied on all spans adjacent to the column under consideration.

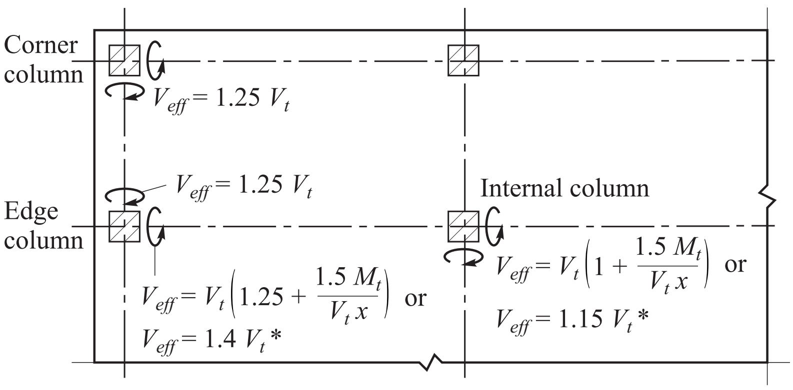


Figure 35: Effective shear forces.

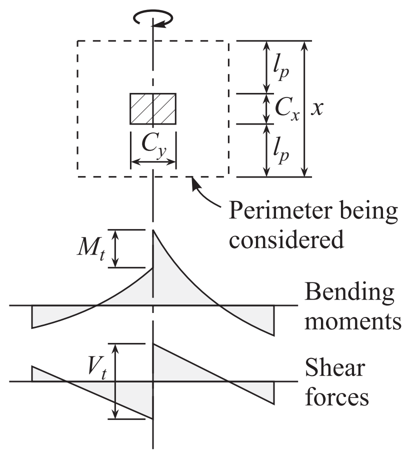


Figure 36: Definition of V_t and M_t.

11 Design of Slender Columns

A vertical load-bearing member is defined as a ***column*** when $h \leq 4b$ and a ***wall*** when $h > 4b$ where h is the larger and b the smaller cross-section dimension, respectively.

11.1 Braced and unbraced columns

A structure can be consider braced if the ratio S_b/S_u is greater than 5, where S_b is the lateral stiffness of the braced structure and S_u is the sway slimness of the unbraced structure.

11.2 Effective lengths

The effective length is determined from:

$$\ell_e = \beta \ell_o \tag{11-1}$$

where ℓ_o = clear height between end restraints and β = factor obtained from Table 28.

Table 28: β-Values for effective lengths.

End condition top	β-Values for effective lengths					
	Braced column			Unbraced column		
	1	2	3	1	2	3
1	0.75	0.80	0.90	1.2	1.3	1.6
2	0.80	0.85	0.95	1.3	1.5	1.8
3	0.90	0.95	1.00	1.6	1.8	-
4	-	-	-	2.2	-	-

Definition of end condition:

1. The end of the column is connected monolithically to beams on either side which are at least **as deep** as the overall dimension of the column in the plane being considered.
2. The end of the column is connected monolithically to beams or slabs on either side which are **shallower** than the overall dimension of the column in the plane being considered.
3. The end of the column is connected to members which, while not designed to provide restraint to rotation, will nevertheless, provide **some restraint**.
4. The end of the column is **unrestrained** against both lateral movement and rotation.

Alternatively, the effective length can be determined from

Braced columns: $$\ell_e = \text{lesser of} \begin{Bmatrix} \ell_o\left[0.7+0.05\left(\alpha_{c,1}+\alpha_{c,2}\right)\right] \\ \ell_o\left[0.85+0.05\alpha_{c,\min}\right] \end{Bmatrix} \le \ell_o \qquad (11\text{-}2)$$

Unbraced columns: $$\ell_e = \text{lesser of} \begin{cases} \ell_o\left[1.0+0.15\left(\alpha_{c,1}+\alpha_{c,2}\right)\right] \\ \ell_o\left[2.0+0.3\alpha_{c,\min}\right] \end{cases} \qquad (11\text{-}3)$$

where $\alpha_{c,1}$ = ratio of the sum of the column stiffness to the sum of the beam stiffnesses at the **lower** end of the column

$\alpha_{c,2}$ = ratio of the sum of the column stiffness to the sum of the beam stiffnesses at the **upper** end of the column

$\alpha_{c,\min}$ = lesser of $\alpha_{c,1}$ and $\alpha_{c,2}$

If a base have been designed to resist the moment, α_c can taken as 1, otherwise α_c should be taken as 10. For a very large stiff base, α_c can taken as 0. For simply supported beams framing into a column α_c should be taken as 10.

11.3 Slenderness

A column should be considered slender when

$$\frac{l_e}{h} > \begin{cases} 17 - 7M_1/M_2 & \text{for braced columns} \\ 10 & \text{for unbraced columns} \end{cases} \qquad (11\text{-}4)$$

where l_e = effective height = $\beta\, l_o$ (β from Table 28)

l_o = clear height between end restraints

M_1 = smaller initial end moment due to ultimate design moments (negative for bending in double curvature)

M_2 = larger initial end moment due to ultimate design moments

Limits: Braced column: $l_o < 60b$ and $b \ge 0.25h$ (11-5)
Unbraced column: $l_o < 25b$ and $b \ge 0.25h$ (11-6)

11.4 Moments and forces in columns

11.4.1 Minimum eccentricity

All columns should therefore be designed for a minimum moment resulting from eccentric loading

$$M_{min} = N\, e_{min} \qquad (11\text{-}7)$$

where $$e_{min} = \begin{Bmatrix} 0.05h & \text{for bending about the } x\text{-axis} \\ 0.05b & \text{for bending about the } y\text{-axis} \end{Bmatrix} \le 20 \text{ mm} \qquad (11\text{-}8)$$

11.4.2 Additional moments in slender columns

$$M_{add} = N\, a_u \tag{11-9}$$

$$a_u = \beta_a\, K\, h \tag{11-10}$$

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{h}\right)^2 \tag{11-11}$$

$$K = \begin{cases} \dfrac{N_{uz} - N}{N_{uz} - N_{bal}} & \text{vir } K \leq 1 \\ \dfrac{M_i}{M_{bal}} & \text{vir } K > 1 \end{cases} \tag{11-12}$$

$$N_{uz} = 0.45\, f_{cu}\, A_c + 0.75\, f_y\, A_{sc} \ (\gamma_m \text{ included}) \tag{11-13}$$

For symmetrically reinforced rectangular sections

$$M_{bal} = 0.046\, f_{cu}\, b\, d^2 + 0.87\, f_{yc}\, A_{sc}\,(d - d') \tag{11-14}$$

$$N_{bal} = 0.25\, f_{cu}\, b\, d \tag{11-15}$$

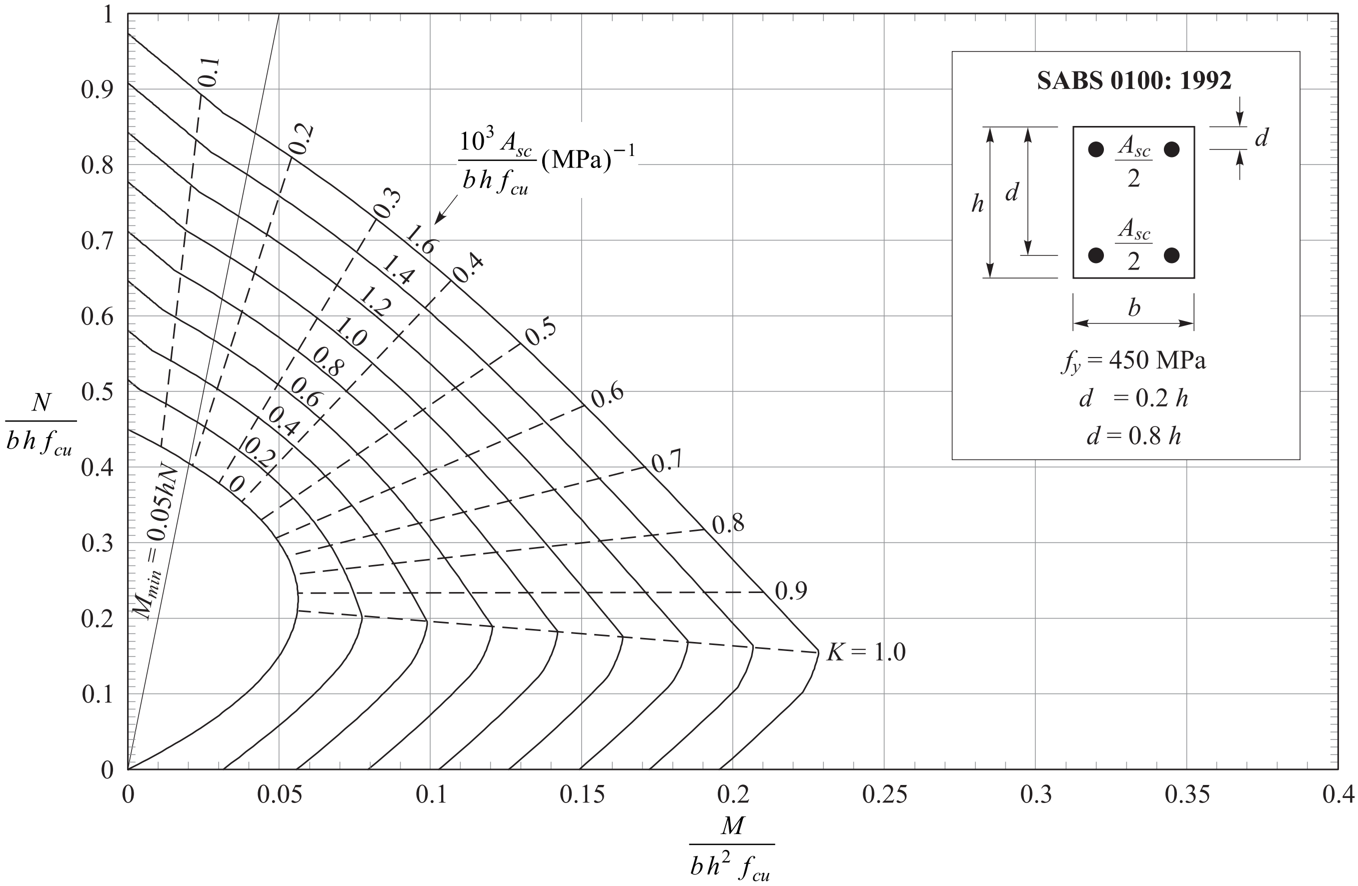


Figure 37: Interaction diagram with K-values.

11.4.3 Braced slender columns

The design moment is the greater of

(a) M_2

(b) $M_i + M_{add}$ (11-16a b c)

(c) $N\,e_{min}$

where $M_i = 0.4\,M_1 + 0.6\,M_2$ (11-17)

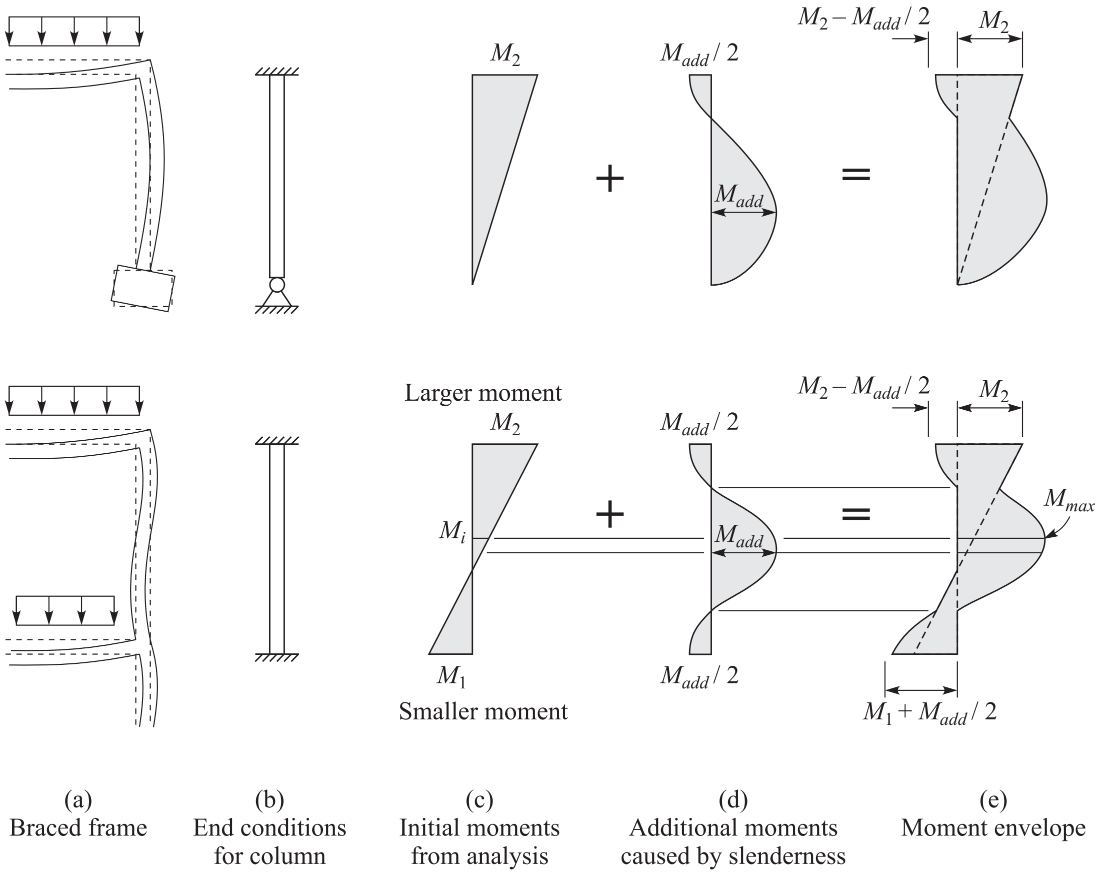


Figure 38: Moments in braced slender columns.

11.4.4 Unbraced slender columns

The design moment is the greater of

(a) $M_2 = M_V + M_H\left(1 + \dfrac{M_{add,unbr}}{M_V + M_H}\right)$

(b) $0.6M_2 + 0.4M_1 + M_{add,braced}$ (11-18a b c)

(c) $N\,e_{min}$

where

$M_{add,braced}$ = additional moment from Eq. (11-3), but using the braced effective length in Eq. (11-5)

M_1, M_2 = smaller and larger end moment including the effects of sway

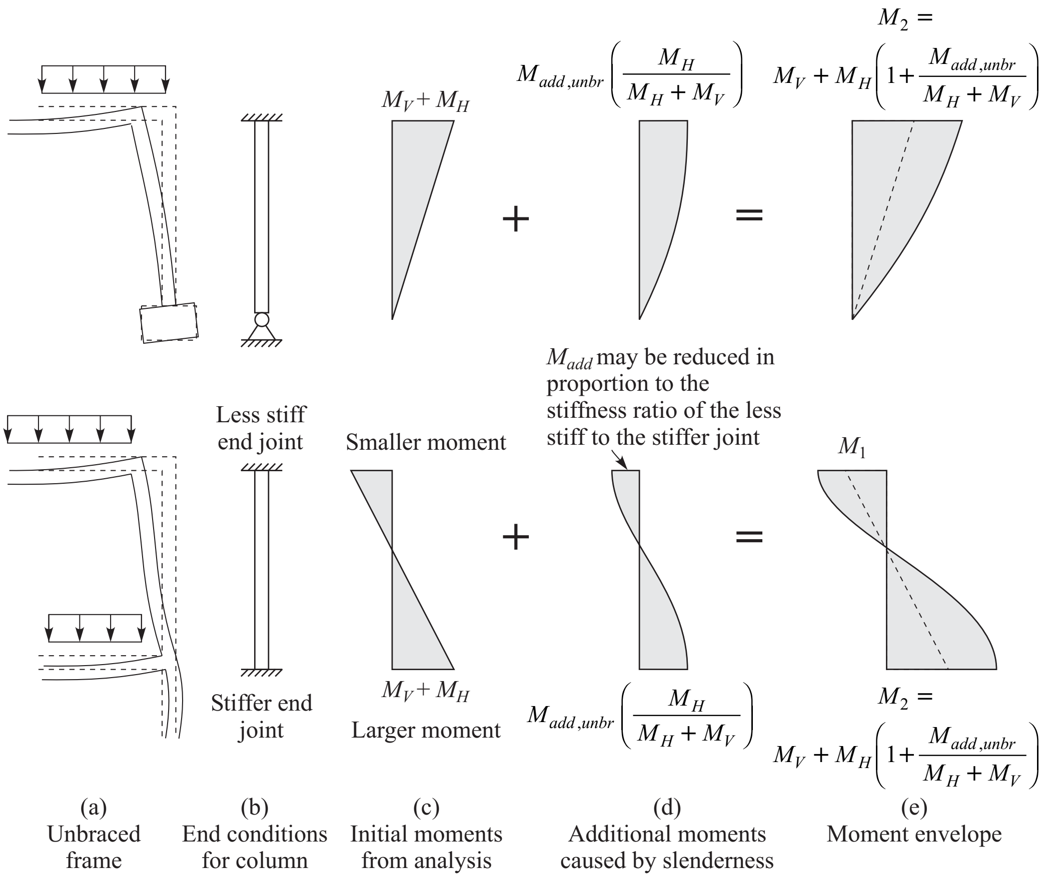


Figure 39: Moments in unbraced slender columns (SABS 0100).

11.4.5 Slender columns bent uniaxially

A column which is slender about both axes but is bent uniaxially, must be designed to resist the additional moment about both axes separately. If the column is slender about one axis only, the additional moment only have to be considered in one plane. Ensure that $M_i \geq M_{min}$.

11.5 Bi-axial bending

$$\text{For } \frac{M_x}{h} > \frac{M_y}{b} \text{ then } M'_x = M_x + \beta_b \frac{h}{b} M_y \qquad (11\text{-}19a\ b)$$

$$\text{For } \frac{M_x}{h} < \frac{M_y}{b} \text{ then } M'_y = M_y + \beta_b \frac{b}{h} M_x$$

where β_b is given in Table 29.

Table 29: β_b-Values for bi-axial bending.

$\frac{N}{bhf_{cu}}$	β_b
0.000	0.50
0.075	0.60
0.150	0.70
0.250	0.70
0.300	0.65
0.400	0.53
0.500	0.42
≥ 0.600	0.30

12 Staircases

The unit weight of the waist, measured horizontally, is determined by multiplying the unit weight measured along the slope of the stair with

$$\frac{\sqrt{R^2+G^2}}{G} \tag{12-1}$$

The unit weight of the stairs (without the waist), measured horizontally, is determined by approximating it as a slab with thickness $R/2$.

If a stair spanning in the direction of the flight is built in at least 110 mm into a wall along the length, a width of 150 mm adjacent to the wall may be deducted from the loaded area. The effective width of the stair may then include 2/3 of the embedded width, up to a maximum of 80 mm.

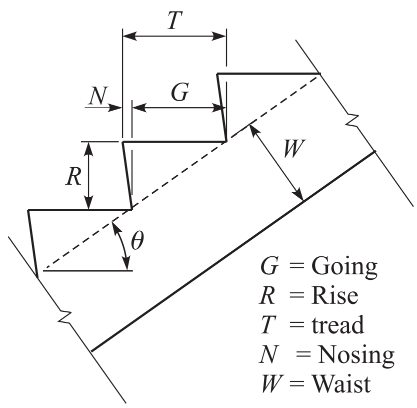


Figure 40: Definition of terms

If a stair is supported by elements spanning at right angels to the span of the stair, the effective span of the stair may be taken as the clear distance between supports plus half the width of the supporting elements, up to a maximum distance of 900 mm at both ends.

The allowable ℓ/d may be increased by 15% if the stairs make out 60% or more of the span.

13 Deflections and Crack Widths

Deflections are calculated from

$$\Delta_i = K M_s \frac{\ell^2}{E_c I_e} \tag{13-1}$$

where
$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \quad \leq I_g \tag{13-2}$$

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{13-3}$$

$$f_r = \begin{cases} 0.65\sqrt{f_{cu}} & \text{for unrestrained sections} \\ 0.30\sqrt{f_{cu}} & \text{for restrained sections where cracking can occur before loading} \end{cases} \tag{13-4}$$

Values for K are given in Figure 41. The total deflection (elastic and creep) is given by

$$\Delta_\infty = \lambda \Delta_i \tag{13-5}$$

where $\lambda = 1 + x_i \phi$ and $x_i = x/d$ (13-6)

If compression reinforcement is present, replace ϕ with

$$\phi' = \phi(1-\rho/2) \tag{13-7}$$

where $\rho = A'_s / A_s$

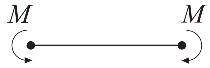
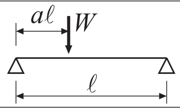
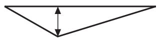
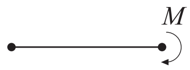
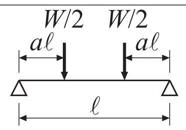
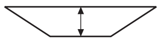
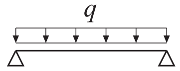
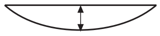
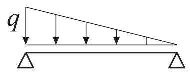
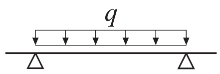
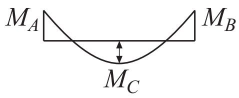
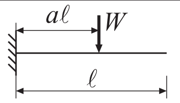
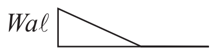
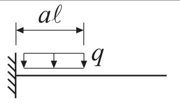
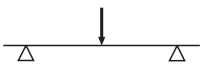
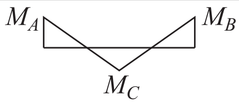
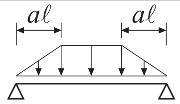
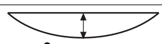
Loading	Bending moment diagram	K
		0.125
	 $M = Wa(1-a)\ell$	$\dfrac{3-4a^2}{48(1-a)}$ If $a = \dfrac{1}{2}$, $K = \dfrac{1}{12}$
		0.0625
	 $M = \dfrac{Wa\ell}{2}$	$0.125 - \dfrac{a^2}{6}$
	 $\dfrac{q\ell^2}{8}$	0.104
	 $\dfrac{q\ell^2}{15.6}$	0.102
		$K = 0.104\left(1 - \dfrac{\beta}{10}\right)$ and $\beta = \dfrac{M_A + M_B}{M_C}$
		End deflection $K = \dfrac{a(3-a)}{6}$ If $a = 1$, $K = 0.333$
	$\dfrac{qa^2\ell^2}{2}$ 	End deflection $K = \dfrac{a(4-a)}{12}$ If $a = 1$, $K = 0.25$
		$K = 0.083\left(1 - \dfrac{\beta}{4}\right)$ $\beta = \dfrac{M_A + M_B}{M_C}$
	 $\dfrac{W\ell^2}{24}(3-4a^2)$	$\dfrac{1}{80}\dfrac{(5-4a^2)^2}{3-4a^2}$

Figure 41: Values for K for different bending moment diagrams.

Deflections caused by shrinkage are determined from

$$\Delta_s = K_s K_{cs} \frac{\varepsilon_s \ell^2}{h} \tag{13-8}$$

where

$$K_s = \begin{cases} \frac{1}{2} & \text{for cantilevers} \\ \frac{1}{8} & \text{for simply supported members} \\ 0.086 & \text{for one end continuous} \\ \frac{1}{16} & \text{for both ends continuous} \end{cases} \tag{13-9}$$

$$K_{cs} = \begin{cases} 0.7\sqrt{\rho\left(1-\dfrac{\rho'}{\rho}\right)} \leq 1 \text{ en } \geq 0 \text{ for uncracked members} \\ 1-\dfrac{\rho'}{\rho}\left[1-0.11(3-\rho)^2\right] \leq 1 \text{ en } \geq 0.3 \text{ for cracked members} \end{cases} \tag{13-10}$$

$$\rho = \frac{100A_s}{bd} \leq 3$$

$$\rho' = \frac{100A_s'}{bd}$$

$$\frac{\rho'}{\rho} \leq 1$$

The maximum crack width is determined from

$$w_{max} = \frac{3\,a_{cr}\varepsilon_m}{1+2\left(\dfrac{a_{cr}-c_{min}}{h-x}\right)} \tag{13-11}$$

where $$\varepsilon_m = \varepsilon_1 - \frac{b_t(h-x)(a'-x)}{3E_sA_s(d-x)} \tag{13-12}$$

ε_1 = concrete strain at the level under consideration

a' = distance from the compression edge to the level under consideration

a_{cr} = distance from the point under consideration to the nearest longitudinal bar

b_t = width of the section at the level of the tension reinforcement

c_{min} = minimum cover to tension reinforcement

To calculate x and ε_1 the cracked transformed section should be based on $E_c/2$ to account for the effects of creep in the concrete.

14 Prestressed Concrete

14.1 Sign convention

- Tensile stresses and forces are positive.
- Hogging bending (concave curvature) is positive.
- The eccentricity e of the prestressing force is measured from the centroid of the section and is taken positive below the centroidal axis.
- The sign of the section modulus $Z = I/y$ with respect to a particular fibre is determined by the distance y of the fibre measured from the centroidal axis. This distance is taken positive for fibres located below the centroidal axis.

14.2 Material properties

Modulus of elasticity:

$$E_p = 205 \text{ GPa for high tensile steel wire} \quad (14\text{-}1)$$
$$= 195 \text{ GPa for 7-wire strand}$$
$$= 165 \text{ GPa for high tensile alloy bars.}$$

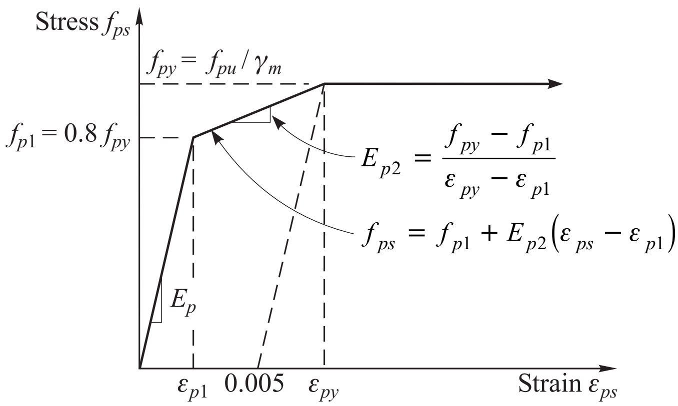


Figure 42: Stress-strain relationship for prestressing steel.

14.3 Elastic stresses

Concrete stresses $$f = \frac{P}{A} + \frac{P\,e\,y}{I} + \frac{M\,y}{I} = \frac{P}{A} + \frac{P\,e}{Z} + \frac{M}{Z} \quad (14\text{-}2)$$

14.4 Ultimate limit state

Strain in prestressed reinforcement

$$\varepsilon_{ps} = \varepsilon_s - \varepsilon_{ce} + \varepsilon_{se} \quad (14\text{-}2)$$

where
$$\varepsilon_{se} = \frac{f_{se}}{E_p} \quad (14\text{-}3)$$

$$\varepsilon_{ce} = \left[\frac{P}{A} + \frac{P e^2}{I}\right]\frac{1}{E_c} \quad (14\text{-}4)$$

$$\varepsilon_s = \left(\frac{d-x}{x}\right)\left|\varepsilon_{cu}\right| \quad (14\text{-}5)$$

Equilibrium (rectangular section)

$$\begin{aligned} F_{cc} &= F_p \\ 0.45\, f_{cu}\, b s &= f_{ps}\, A_{ps} \end{aligned} \quad (14\text{-}6)$$

$$M_u = F_{cc}\, z = F_{ps}\, z \quad (14\text{-}7)$$

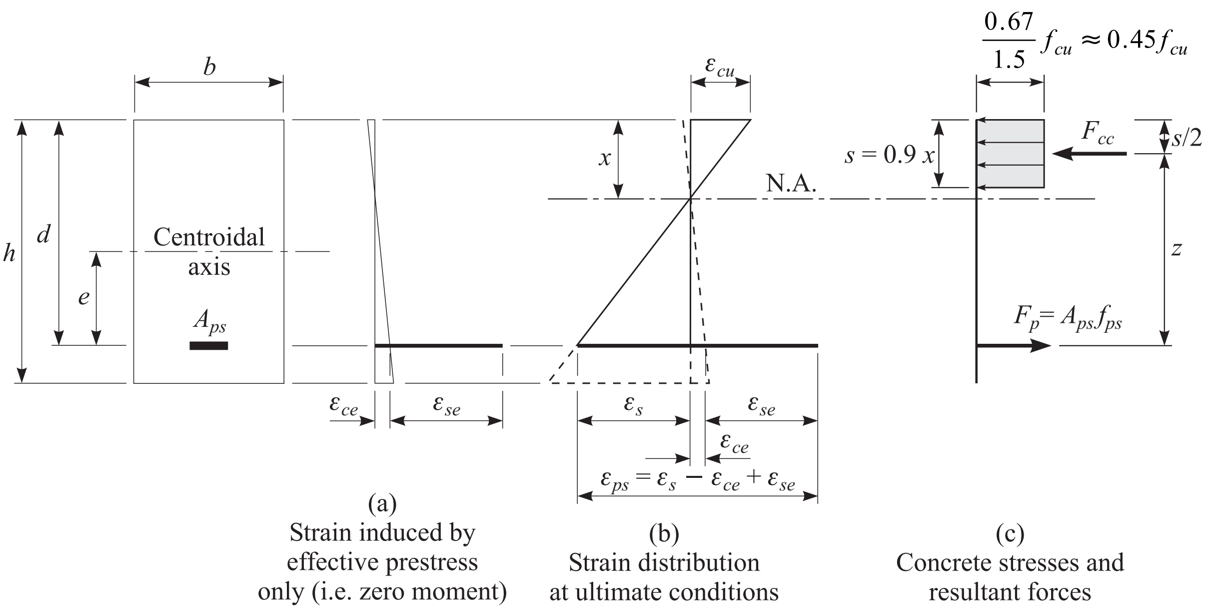


Figure 43: Prestressed concrete beam at ultimate.

Table 30: Reinforcement areas (mm^2/m).

Spacing (mm)	Bar diameter (mm)							
	8	10	12	16	20	25	32	40
1000	50	79	113	201	314	491	804	1257
975	52	81	116	206	322	503	825	1289
950	53	83	119	212	331	517	847	1323
925	54	85	122	217	340	531	869	1359
900	56	87	126	223	349	545	894	1396
875	57	90	129	230	359	561	919	1436
850	59	92	133	237	370	577	946	1478
825	61	95	137	244	381	595	975	1523
800	63	98	141	251	393	614	1005	1571
775	65	101	146	259	405	633	1038	1621
750	67	105	151	268	419	654	1072	1676
725	69	108	156	277	433	677	1109	1733
700	72	112	162	287	449	701	1149	1795
675	74	116	168	298	465	727	1191	1862
650	77	121	174	309	483	755	1237	1933
625	80	126	181	322	503	785	1287	2011
600	84	131	188	335	524	818	1340	2094
575	87	137	197	350	546	854	1399	2185
550	91	143	206	366	571	892	1462	2285
525	96	150	215	383	598	935	1532	2394
500	101	157	226	402	628	982	1608	2513
475	106	165	238	423	661	1033	1693	2646
450	112	175	251	447	698	1091	1787	2793
425	118	185	266	473	739	1155	1892	2957
400	126	196	283	503	785	1227	2011	3142
375	134	209	302	536	838	1309	2145	3351
350	144	224	323	574	898	1402	2298	3590
325	155	242	348	619	967	1510	2475	3867
300	168	262	377	670	1047	1636	2681	4189
275	183	286	411	731	1142	1785	2925	4570
250	201	314	452	804	1257	1963	3217	5027
225	223	349	503	894	1396	2182	3574	5585
200	251	393	565	1005	1571	2454	4021	6283
175	287	449	646	1149	1795	2805	4596	7181
150	335	524	754	1340	2094	3272	5362	8378
125	402	628	905	1608	2513	3927	6434	10053
100	503	785	1131	2011	3142	4909	8042	12566
75	670	1047	1508	2681	4189	6545	10723	16755
50	1005	1571	2262	4021	6283	9817	16085	25133
25	2011	3142	4524	8042	12566	19635	32170	50265

Table 31: Reinforcement areas (mm^2).

No	Bar diameter (mm)							
	8	10	12	16	20	25	32	40
1	50.3	78.5	113.1	201.1	314.2	490.9	804.2	1256.6
2	101	157	226	402	628	982	1608	2513
3	151	236	339	603	942	1473	2413	3770
4	201	314	452	804	1257	1963	3217	5027
5	251	393	565	1005	1571	2454	4021	6283
6	302	471	679	1206	1885	2945	4825	7540
7	352	550	792	1407	2199	3436	5630	8796
8	402	628	905	1608	2513	3927	6434	10053
9	452	707	1018	1810	2827	4418	7238	11310
10	503	785	1131	2011	3142	4909	8042	12566
11	553	864	1244	2212	3456	5400	8847	13823
12	603	942	1357	2413	3770	5890	9651	15080
13	653	1021	1470	2614	4084	6381	10455	16336
14	704	1100	1583	2815	4398	6872	11259	17593
15	754	1178	1696	3016	4712	7363	12064	18850
16	804	1257	1810	3217	5027	7854	12868	20106
17	855	1335	1923	3418	5341	8345	13672	21363
18	905	1414	2036	3619	5655	8836	14476	22619
19	955	1492	2149	3820	5969	9327	15281	23876
20	1005	1571	2262	4021	6283	9817	16085	25133
21	1056	1649	2375	4222	6597	10308	16889	26389
22	1106	1728	2488	4423	6912	10799	17693	27646
23	1156	1806	2601	4624	7226	11290	18498	28903
24	1206	1885	2714	4825	7540	11781	19302	30159
25	1257	1963	2827	5027	7854	12272	20106	31416
26	1307	2042	2941	5228	8168	12763	20910	32673
27	1357	2121	3054	5429	8482	13254	21715	33929
28	1407	2199	3167	5630	8796	13744	22519	35186
29	1458	2278	3280	5831	9111	14235	23323	36442
30	1508	2356	3393	6032	9425	14726	24127	37699
31	1558	2435	3506	6233	9739	15217	24932	38956
32	1608	2513	3619	6434	10053	15708	25736	40212
33	1659	2592	3732	6635	10367	16199	26540	41469
34	1709	2670	3845	6836	10681	16690	27344	42726
35	1759	2749	3958	7037	10996	17181	28149	43982
36	1810	2827	4072	7238	11310	17671	28953	45239
37	1860	2906	4185	7439	11624	18162	29757	46496
38	1910	2985	4298	7640	11938	18653	30561	47752
39	1960	3063	4411	7841	12252	19144	31366	49009
40	2011	3142	4524	8042	12566	19635	32170	50265